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Using Bayesian Statistics in Enterprise Demography*

Abstract

Knowledge of the number of different kinds of enterprises that will be created in a coming year is essential information. It can be used in macroeconomic analyses and as a constituent of the background for economic policy.

From a demographics point of view, we consider the creation (birth) of some enterprise as a basic indicator. It can also be approached from the point of view of inference, as the creation of enterprise is influenced by a wide variety of inputs. Enterprise creation may therefore be thought of as a random process.

The analytic tools Bayesian statistics provide make it possible involve more kinds of information into statistical analysis and gradually update the parameter estimations. We used the conjugate family Poisson / gamma to estimate the number of enterprises to be created in a coming year. The considerations were concerned with the mean square error, which was used as the main criterion of the point estimation quality. We solved two kinds of problems: to find a Bayesian point estimation that has a smaller mean square error than the classical one in a predetermined interval, and, along with it, to model prior information in a very simple way.

In finding some connection among the variables contained in the conjugate family Poisson / gamma, we solved both presented problems and also developed a simple algo-

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rithm for optimal point estimation of the Poisson distribution parameter. This algorithm was used to estimate the number of enterprises created.

Keywords: Bayesian point estimation, mean square error, conjugate family, prior distribution, posterior distribution, number of enterprise births.

1. Introduction

The Bayesian approach is very useful in statistical analysis whenever there is a lack of reliable information. Statistical inference is a suitable tool for solving problems when the random sample is large enough, so that the inference conclusions derived from the information provided by it are credible. Sometimes, along with the random sample, other information is available about a particular indicator or estimated parameter we are dealing with; in such cases it is advisable to use the Bayesian approach, which enables techniques and algorithms for including two (or more) sources of information into a statistical analysis. Employing more information leads to more qualitative and more credible conclusions. The fundamental advantage of using the Bayesian approach is that it yields more precise results: point estimation has better properties and a narrower confidence interval.

The main disadvantage of the Bayesian approach, on the other hand, is that it is based on more difficult mathematics. That is probably the main reason it is not used in practice as widely as might be desirable. One field that does use it is the insurance industry, particularly to estimate insurance event probability, the number of insurance events and the average insurance cover (Pacáková 2004). The empirical credible theory was developed on the basis of Bayesian theory (Šoltés 2009).

This article examines the point estimation of the number of enterprises that are going to be established in some future period of time. The enterprises are categorised by type of economic activity they engage in and their number of employees. The process of creating an enterprise is influenced by a wide variety of factors, so it may be considered a random event. The number of enterprises that are going to be established in the next year is an indicator worth estimating.

The purpose of the article is to introduce the original approach of creating Bayesian point estimation and to apply an algorithm developed to estimate the number of enterprise births.

2. The Principle of Bayesian Statistics

Bayesian statistics connect and utilise two kinds of information: random sample and, along with it, prior information (Lee 2012) which comes from another source.

In comparison with classical statistical inference, Bayesian statistics requires more rigorous mathematics and is characterised by a higher level of abstraction. The estimated parameter is considered a random variable, the distribution of which is updated by including the data that arises from a random sample. The prior information, which is usually available before the data from a survey, is created by so-called prior distribution. Including the data from random sampling leads to the posterior distribution, on the basis of which the inference conclusions are made.

As the posterior distributions' variance is smaller than both the sample variance and the prior variance, the confidence intervals obtained are narrower than those the classical approach yields. The difference between the ranges is considerable, especially when the posterior density is not symmetric – the highest posterior density region, which is used in Bayesian statistics for interval estimation, is much more precise (Bernardo & Smith 2000, Bolstad 2004, Garthwaite, Jolliffe & Jones 2002).

The theory of Bayesian statistics is based on Bayes' theorem, the continuous form of which is written:

$$f_\Theta(\theta \mid \mathbf{x}) = \frac{f(\mathbf{x} \mid \theta) \cdot f_\Theta(\theta)}{\int_\Theta f(\mathbf{x} \mid \theta) \cdot f_\Theta(\theta)\, \mathbf{d}\theta}, \tag{1}$$

where:

$f_\Theta(\theta)$ denotes the prior density of the estimated parameter Θ,

$f_\Theta(\theta \mid \mathbf{x})$ denotes the posterior density of Θ,

$f(\mathbf{x} \mid \theta)$ denotes the likelihood function.

The connections among the distributions, along with some other information, are derived from the simplified form of Bayes' theorem, in which the equation is substituted with the proportion:

$$f_\Theta(\theta \mid \mathbf{x}) \propto f(\mathbf{x} \mid \theta) \cdot f_\Theta(\theta). \tag{2}$$

When the prior and the posterior are of the same type, they are called conjugated distribution in relation to the sample distribution. The three distributions (prior, posterior and sample) create what is called a conjugate family[1]. Here the definition of posterior is very simple as formulas exist for evaluating the posteriors' parameters (the values of prior parameters and some sample characteristics are substituted).

In practice, three conjugated families are commonly used (Kotlebová 2009, Pacáková *et al*. 2012):

[1] In (Weerahandi 1995) it is called "natural conjugate family of distributions for the distribution of variable X".

– binomial/beta – for estimating the binomial proportion π,
– Poisson/gamma – for estimating the Poisson mean λ,
– Normal/normal – for estimating the normal mean μ.

The Bayesian point estimation of some parameter Θ is usually the posterior mean, but sometimes (depending on the type of loss function) it may be the distribution's median or mode, too (Pacáková 2004).

In this article, we take a detailed look at the second of the listed conjugated families. It was used to estimate the number of enterprises to be created, depending on their activities and number of employees.

In conjugate family Poisson/gamma, the sample distribution is Poisson distribution, and the prior distribution of its parameter λ is gamma distribution $G(\alpha; \beta)$.

Then the posterior of λ (denoted $\lambda/\mathbf{x}$) is gamma distribution $G(\alpha'; \beta')$, too. The parameters $\alpha'; \beta'$ satisfy: $\alpha' = \alpha + \sum_{i=1}^{n} x_i$, while $\beta' = \beta + n$ $(x_1, x_2, \ldots, x_n) = \mathbf{x}$ is random sample data (Kotlebová 2009).

We adopted the mean square error as the criterion for the point estimation quality. A similar theory was developed for the conjugated family binomial/beta (see Kotlebová & Láska 2014a, 2014b for possible applications).

3. Properties of the Point Estimations – The Mean Square Error

The point estimation of a distribution parameter θ is the sample characteristic Un ($est\,\Theta = U_n$), which satisfies certain conditions. It has to be:

– unbiased – its mean must be equal to the estimated parameter ($E(U_n) = \Theta$);
– consistent – increasing the sample size makes the estimation more precise (its value is closer to the estimated parameter);
– efficient – its variance is the smallest among the variances of all unbiased estimators[2].

Among these properties, primacy is afforded the first, as it is the necessary condition for efficiency (consistency, too, is somewhat dependent). If the estimator is only slightly biased, it cannot be efficient, too. Thus, an estimator with large variance may be preferred against a slightly biased estimator with low variance – it is obvious that a little bias is better than huge variance in the unbiased estimator.

The sensible way to fairly take into account both properties is to consider the mean square error – the sum of variance and the square of bias (Wonnacott & Wonnacott 1990):

[2] In addition to these properties, sufficiency and robustness are usually presented (Pacáková *et al*. 2012).

$$MSE(U_n) = E\left[(\Theta - U_n)^2\right] = D(U_n) + \Delta_n^2, \tag{3}$$

where $D(U_n)$ is the variance of U_n and $\Delta_n = E(U_n) - \Theta$ is the bias.

According to this criterion, the better estimator is the one with the smaller mean square error. We were looking for the Bayesian point estimator with smaller mean square error in comparison with the classical point estimation. Along with it, we developed an algorithm that optimally determines the prior parameters' values based on a simple conception of parameter's value.

Kotlebová and Láska (2014a, 2014) showed, for the conjugate family binomial/beta, that if, according to the prior concept, the estimated parameter π is within some particular interval, it is possible to create a prior distribution that will lead to a posterior that gives a Bayesian point estimation with a smaller mean square error than the classical point estimation just within this interval.

4. Bayesian Point Estimation of the Poisson Mean

As mentioned earlier, the conjugate family Poisson/gamma is convenient for inference conclusions of the Poisson mean. In addition to being rather simple, gamma distribution is flexible enough to shape a prior conception by setting convenient parameter values.

To make the following considerations clear, we shall once again go over the conjugate family we are to deal with:

If the sample distribution is Poisson distribution and the prior distribution of parameter λ is gamma distribution $G(\alpha; \beta)$, the posterior distribution is also gamma distribution, with parameters: $\alpha' = \alpha + \sum_{i=1}^{n} x_i$, $\beta' = \beta + n$.

The classical point estimation of parameter λ is the sample mean: $est\lambda = \bar{x}$, an unbiased estimator whose mean square error takes the form:

$$MSE(\bar{X}) = D(\bar{X}) + 0^2 = \frac{\lambda}{n}. \tag{4}$$

(Considering n as a constant, we may think of the mean square error as a linear function of independent variable λ).

Bayesian point estimation of λ (denoted by $\hat{\lambda}_B$) is the posterior distributions' $G\left(\alpha + \sum_{i=1}^{n} x_i; \beta + n\right)$ mean:

$$\hat{\lambda}_B = E(\lambda/\mathbf{x}) = \frac{\alpha + n\bar{x}}{\beta + n}. \tag{5}$$

To express the mean square error of the Bayesian point estimation, we need its variance and (potential) bias.

The posterior mean (Bayesian point estimation) may be expressed as follows:

$$E(\hat{\lambda}_B)=\frac{1}{\beta+n}\left[E(\alpha)+nE(\bar{X})\right]=\frac{\alpha+n\lambda}{\beta+n}\neq\lambda. \tag{6}$$

It is obvious that the estimator is not unbiased. The bias is:

$$\frac{\alpha+n\lambda}{\beta+n}-\lambda=\frac{\alpha-\beta\lambda}{\beta+n}. \tag{7}$$

The variance of the Bayesian point estimation is expressed as:

$$D(\hat{\lambda}_B)=\frac{1}{(\beta+n)^2}\left[D(\alpha)+n^2D(\bar{X})\right]=\frac{1}{(\beta+n)^2}[0+n\lambda]=\frac{n\lambda}{(\beta+n)^2}. \tag{8}$$

(As $\frac{n\lambda}{(\beta+n)^2}<\frac{\lambda}{n}$, the Bayesian point estimation has a smaller variance than the classical point estimation of λ).

Thus, the mean square error may be expressed as follows:

$$MSE(\hat{\lambda}_B)=\frac{1}{(\beta+n)^2}[(\alpha-\beta\lambda)^2+n\lambda]=\frac{1}{(\beta+n)^2}[\beta^2\lambda^2+\lambda(n-2\alpha\beta)+\alpha^2]. \tag{9}$$

Considering this expression as a function of variable λ, it should be recognised as a convex quadratic function, which when graphed does not intersect the *x* axis.

We were looking for an interval within which the mean square error of the Bayesian point estimation is smaller than the mean square error of the classical point estimation. If, theoretically, the prior mean equalled $\lambda\left(\lambda=\frac{\alpha}{\beta}\right)$, the required condition would be satisfied:

$$\begin{aligned}MSE(\hat{\lambda}_B)&=\frac{1}{(\beta+n)^2}\left[\left(\alpha-\beta\cdot\frac{\alpha}{\beta}\right)^2+n\frac{\alpha}{\beta}\right]=\frac{n\alpha}{\beta(\beta+n)^2}=\\&=\frac{\alpha}{\beta n}\cdot\frac{n^2}{(\beta+n)^2}<\frac{\alpha}{\beta n}\left(=MSE(\bar{X})\text{ for }\lambda=\frac{\alpha}{\beta}\right).\end{aligned} \tag{10}$$

But this assumption is made up expressly to show that: If there exists some point in which the graph of quadratic function is below the graph of the linear function, there must exist some interval (containing the point) $(\lambda_1;\lambda_2)$ within which the condition $\left(MSE(\hat{\lambda}_B)<MSE(\bar{X})\right)$ is also satisfied.

We tried to find some connection between the interval and the prior distribution. The goal was to determine such values of the prior distributions' parameters which would lead to the Bayesian point estimation with the smaller mean square error (compared to classical point estimation) just at interval $(\lambda_1;\lambda_2)$.

To find the connection between the variables listed, the following system of equations must be solved:

$$\frac{1}{(\beta+n)^2}[(\alpha-\beta\lambda_1)^2+n\lambda_1]=\frac{\lambda_1}{n}, \tag{11}$$

$$\frac{1}{(\beta+n)^2}\left[(\alpha-\beta\lambda_2)^2+n\lambda_2\right]=\frac{\lambda_2}{n}. \tag{12}$$

The solution is:

$$\beta_{12}=\frac{4n\left[n(\lambda_1+\lambda_2)-1\right]\pm 8n^2\sqrt{\lambda_1\lambda_2}}{2\left[n^2(\lambda_2-\lambda_1)^2-2n(\lambda_1+\lambda_2)+1\right]}=\frac{2n\left[n(\lambda_1+\lambda_2)-1\right]\pm 4n^2\sqrt{\lambda_1\lambda_2}}{\left[n^2(\lambda_2-\lambda_1)^2-2n(\lambda_1+\lambda_2)+1\right]}, \tag{13}$$

$$\alpha=\frac{1}{2n}\left[\beta n(\lambda_1+\lambda_2)-\beta-2n\right]. \tag{14}$$

As can be seen, there are two solutions, but only one of them solves the problem: if in the expression for β we choose the possibility "–", then variable α is negative. So, the prior parameters are these:

$$\beta=\frac{2n\left[n(\lambda_1+\lambda_2)-1\right]+4n^2\sqrt{\lambda_1\lambda_2}}{\left[n^2(\lambda_2-\lambda_1)^2-2n(\lambda_1+\lambda_2)+1\right]}, \tag{15}$$

$$\alpha=\frac{1}{2n}\left[\beta n(\lambda_1+\lambda_2)-\beta-2n\right]. \tag{16}$$

The result we arrived at is well applicable in practice, as finding the optimal prior is one weakness of the Bayesian approach. Usually, the mean can be evaluated quite exactly, but visualising variance is not so simple. (If we were sure of the values of both parameters, we would determine the prior parameters by solving this system of equations: $E(\lambda)=\frac{\alpha}{\beta}$; $D(\lambda)=\frac{\alpha}{\beta^2}$).

More simply, the prior concept would be expressed by borders between which the estimated parameter is placed. That means that there exists an interval $(\lambda_1;\lambda_2)$ containing λ according to a prior belief. Such a concept may be expressed by anybody (it is not necessary to understand the principle of Bayesian statistics).

Thus, if we knew the borders of the interval containing the estimated parameter, using (15) and (16) we would evaluate parameters of such prior distribution so that the Bayesian point estimation based on it would be superior to the classical one in terms of the smaller mean square error.

Here is an example: Take for the variables the values $n = 20$, $\lambda_1 = 6$, $\lambda_2 = 10$. Substituting these into (15) and (16) would yield the prior distributions parameters: $\alpha = 33.820263$, $\beta = 4.366177$. Meanwhile, the mean square errors of classical and Bayesian estimation may be expressed as functions according to (4) and (9):

$$MSE(\lambda)=\frac{\lambda}{n}=\frac{\lambda}{20},$$

$$MSE(\hat{\lambda}_B)=\frac{1}{(\beta+n)^2}\left[\beta^2\lambda^2+\lambda(n-2\alpha\beta)+\alpha^2\right]=$$

$$=\frac{1}{(4{,}366177+20)^2}\cdot\left[4{,}366177^2\cdot\lambda^2+\lambda(20-2\cdot 33{,}820263\cdot 4{,}366177)+33{,}820263^2\right]=$$

$$=0{,}032109\cdot\lambda^2-0{,}463745\cdot\lambda+1{,}924537.$$

The functions are graphed in Fig. 1. We concentrated on the values λ of placed in interval $\langle 5;11 \rangle$.

As can be seen, the intersections of the graphs are in [6; 0.3] and [10; 0.5]. At interval (6; 10) the inequation $MSE(\hat{\lambda}_B) < MSE(\bar{X})$ is valid, outside of interval the reverse inequality is satisfied.

For the three particular values $n = 20$, $\lambda_1 = 6$, $\lambda_2 = 10$ (according to (15) and (16)) the prior distributions' parameters were determined in a way that led to the Bayesian point estimation with the smaller mean square error just at interval (6; 10).

The part of interval $(\lambda_1; \lambda_2)$ in which the Bayesian point estimation is placed depends on variable $\sum_{i=1}^{n} x_i$, which is evaluated from the sample data. (It may sometimes occur that if the sample mean is outside of interval $(\lambda_1; \lambda_2)$, the Bayesian point estimation is, too. This would indicate that the prior concept is far from reality).

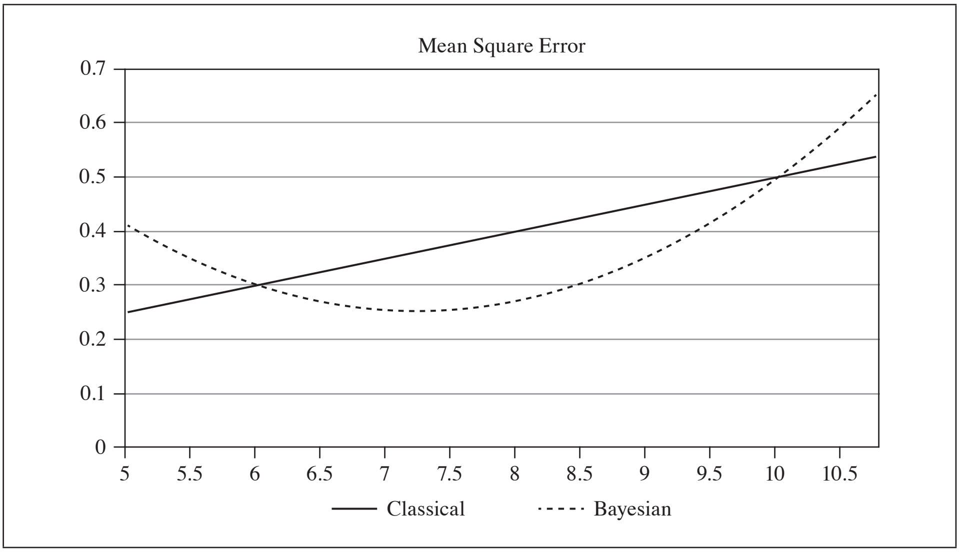

Fig. 1. A Comparing of the Graphs of the Classical and the Bayesian Mean Square Error
Source: the authors' calculations.

The relations mentioned above allowed us to design an algorithm that would determine the qualitative Bayesian point estimation of Poisson mean λ on the basis of the simple prior concept:

1) determining the borders of interval $(\lambda_1; \lambda_2)$, within which parameter λ has to be situated;

2) evaluating the prior distributions' parameters α, β according to the formulas

$$\beta = \frac{2n\left[n(\lambda_1+\lambda_2)-1\right]+4n^2\sqrt{\lambda_1\lambda_2}}{\left[n^2(\lambda_2-\lambda_1)^2-2n(\lambda_1+\lambda_2)+1\right]},\quad \alpha=\frac{1}{2n}\left[\beta n(\lambda_1+\lambda_2)-\beta-2n\right];$$

3) on the basis of observed sample data, evaluating the Bayesian point estimation of λ:

$$\hat{\lambda}_B = E(\lambda/\mathbf{x}) = \frac{\alpha+n\bar{x}}{\beta+n}.$$

We applied the introduced algorithm to estimate the number of enterprises which are going to be created in Slovakia.

5. Estimating the Number of Different Enterprises (according to NACE) Created in the Next Year in Slovakia

Creating an enterprise may be considered a basic demographic event: a birth. However, since this creation is influenced by a wide range of factors, it may also be considered a random event. The number of such events may be modelled by Poisson distribution, which is widely used to estimate the number of insurance events. To estimate the mean of the distribution λ, we had enough information to use the Bayesian approach. Thus, the conjugate family Poisson/gamma was appropriate. In the SLOVSTAT database, the data on creating enterprises according to NACE classification is available for the years 2008 to 2011. The former classification (OKEČ) contains data for the years 2000–2007. Some kinds of economic activity are covered by both classifications. They are listed in Table 1.

Table 1. List of Economic Activities Covered by Both the OKEČ and NACE Databases

Code of Activity		Economic Activity
OKEČ	NACE	
C	B	Mining and quarrying
D	C	Manufacturing
F	F	Construction
H	I	Accommodation and food service activities
J	K	Financial and insurance activities
K	L	Real estate activities
M	P	Education

Source: www.statistics.sk/pls/wregis/ciselniky?kc=5205, accessed: July 2014.

Table 2. Number of Enterprise Births by Economic Activities and Size Class from 2000 to 2011

NACE	Number of Employees	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
B	0–4	4	19	10	3	4	9	4	15	12	27	22	20
	5–9	1	2	1	1	3	2	0	2	0	1	0	2
	10 and more	0	0	0	2	2	3	1	3	0	1	0	2
	total	5	21	11	6	9	14	5	20	12	29	22	24
D	0–4	3494	5753	6832	3400	3779	2995	10331	10541	9864	7986	5650	7500
	5–9	95	115	69	118	131	154	115	260	147	91	67	62
	10 and more	153	187	114	134	127	157	87	271	188	89	67	52
	total	3742	6055	7015	3652	4037	3306	10533	11072	10199	8166	5784	7614
F	0–4	3103	5044	6589	3688	4105	3330	8267	8482	11650	11929	9651	12425
	5–9	45	70	39	63	79	124	48	218	186	94	47	76
	10 and more	49	71	38	82	80	95	23	140	113	50	38	37
	total	3197	5185	6666	3833	4264	3549	8338	8840	11949	12073	9736	12538
I	0–4	1066	1381	1766	1135	1307	992	953	1286	1638	2311	2062	2200
	5–9	36	39	30	79	82	185	36	120	139	54	8	43
	10 and more	32	44	12	38	39	86	25	82	76	24	9	19
	total	1134	1464	1808	1252	1428	1263	1014	1488	1853	2389	2079	2262
K	0–4	89	184	291	65	87	61	1024	582	595	598	420	579
	5–9	5	5	2	2	1	4	4	10	11	3	4	2
	10 and more	1	4	2	5	7	4	5	5	4	1	4	0
	total	95	193	295	72	95	69	1033	597	610	602	428	581

NACE	Number of Employees	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
L	0–4	3439	7168	8354	4144	5821	5011	6605	6952	2287	2155	1774	3473
	5–9	74	124	80	106	139	261	143	299	60	22	17	23
	10 and more	62	109	73	97	120	211	81	260	62	20	12	7
	total	3575	7401	8507	4347	6080	5483	6829	7511	2409	2197	1803	3503
P	0–4	228	342	437	286	467	315	432	373	529	733	615	880
	5–9	1	4	2	6	8	38	27	51	25	16	20	19
	10 and more	0	1	5	4	6	56	21	69	38	31	43	11
	total	229	347	444	296	481	409	480	493	592	780	678	910

Source: http://www.statistics.sk/pls/elisw/objekt.send?uic=3506&m_so=5, accessed: July 2014.

Table 3. Calculation of the Bayesian Point Estimation of Enterprise Births

NACE	Number of Employees	Total 2000–2011	Minimum (λ_1)	Maximum (λ_2)	n	β	α	Bayesian Point Estimation
B	0–4	149	3	27	12	0,167832	1,51049	12,3695
	5–9	15	0	3	12	0,685714	–1,5E-16	1,18243
	10 and more	14	0	3	12	0,685714	–1,5E-16	1,1036
	total	178	5	29	12	0,203387	2,449102	14,78681
D	0–4	78125	2995	10541	12	0,00087	4,88921	6510,35
	5–9	1424	62	260	12	0,029417	3,734936	118,687
	10 and more	1626	52	271	12	0,023393	2,776927	135,467
	total	81175	3306	11072	12	0,000878	5,312573	6764,531

Table 3 cnt'd

NACE	Number of Employees	Total 2000–2011	Minimum (λ_1)	Maximum (λ_2)	n	β	α	Bayesian Point Estimation
F	0–4	88263	3103	12425	12	0,000643	3,993833	7355,19
	5–9	1089	39	218	12	0,027585	2,543478	90,7533
	10 and more	816	23	140	12	0,040464	2,296135	67,9622
	total	90168	3197	12538	12	0,000651	4,121154	7513,936
I	0–4	18097	953	2311	12	0,006761	10,03304	1508,07
	5–9	851	8	185	12	0,017245	0,663431	70,8701
	10 and more	486	9	86	12	0,050923	1,416722	40,4464
	total	19434	1014	2389	12	0,006895	10,73117	1619,464
K	0–4	4575	61	1024	12	0,003418	0,854366	381,213
	5–9	53	1	11	12	0,378543	1,255485	4,38303
	10 and more	42	0	7	12	0,289157	–1,5E-16	3,41765
	total	4670	69	1033	12	0,003521	0,940125	389,1308
L	0–4	57183	1774	8354	12	0,000824	3,170326	4765,19
	5–9	1348	17	299	12	0,011539	0,82267	112,294
	10 and more	1114	7	260	12	0,011014	0,469856	92,7873
	total	59645	1803	8507	12	0,000807	3,162034	4970,346
P	0–4	5637	228	880	12	0,009431	4,224555	469,733
	5–9	217	1	51	12	0,053144	0,379522	18,0351
	10 and more	285	0	69	12	0,029021	1,48E-16	23,6927
	total	6139	229	910	12	0,008853	4,041203	511,5427

Source: the authors' calculations.

For activities listed in Table 1, the longer time series (2000–2011) may be used, while for others only data since 2008 can be used.

Table 2 lists the enterprise births in the SR by economic activity and size class category by number of employees (the period 2000–2011).

Using the data, and the algorithm we have introduced, we estimated the number of enterprise births for the next year 2012. The prior information was created very simply: the minimum number in the whole time series was used as the low border λ_1, while the maximum was the second border λ_2. Done in Excel, the calculations can be found in Table 3.

The values calculated and listed in the individual columns in Table 3 correspond to the algorithm described at the end of the previous section. As may be seen from the calculations, Bayesian point estimation is a number within the interval $(\lambda_1; \lambda_2)$. The longer the available time series, the more precise the estimation will be. In other words, more information improves the quality of the estimation.

6. Conclusions

This article has presented the potentialities of using Bayesian statistics in analyses of the basic indicator in enterprise demography. Inference methods are applied mostly for data taken from a random survey. However, when the event (enterprise birth) is influenced by a number of factors, we may consider it to be a random event and approach it from this point of view.

Bayesian statistics provides an effective tool for sequentially updating some indicators. In the contribution, we have examined the estimation of enterprise births by means of Poisson distribution mean. We used the SLOVSTAT database, which contains the enterprise births in the SR by economic activity and size class by number of employees listed for the years 2000 to 2011.

Although we achieved some factual results, including an estimation of enterprise births for the next period, the value here is mainly theoretical: as a quality criterion of point estimation, we used the mean square error, which we sought to minimise. We examined the connection between a variety of variables within the frame of a conjugate family Poisson/gamma and we succeeded in creating an algorithm that would evaluate the Bayesian point estimation, which has a smaller mean square error than its classical counterpart within the predetermined interval. We consider the ability to create the prior distribution in a very simple way to be important and useful knowledge – it suffices to determine the borders of an interval, within which the estimated parameter has to be placed.

The algorithm we developed was illustrated with an example in which the number of enterprise births was estimated on the basis of data from previous periods.

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Zastosowanie metod statystyki bayesowskiej w demografii przedsiębiorstw

(Streszczenie)

Znajomość liczby przedsiębiorstw różnego typu, których utworzenie jest planowane w najbliższym roku, stanowi istotną informację, która może zostać wykorzystana w aspekcie makroekonomicznym, a także może stanowić podstawę do kreowania polityki ekonomicznej.

Z demograficznego punktu widzenia podstawowym przedmiotem rozważań jest powstanie przedsiębiorstwa. Możliwe jest również podejście nawiązujące do zasad wnioskowania statystycznego, gdyż na tworzenie przedsiębiorstw oddziałują liczne i zróżnicowane czynniki, co daje podstawy do postrzegania tego procesu jako losowego.

Metody analityczne statystyki bayesowskiej dają możliwość uwzględnienia w procesie badania większej ilości informacji oraz stopniowej korekty oszacowania danego parametru.

Do oszacowania liczby planowanych do utworzenia przedsiębiorstw wykorzystano rodzinę rozkładów sprzężonych Poisson-gamma. Niezbędne rozważania oparte zostały na błędzie średniokwadratowym, przyjętym jako główne kryterium oceny jakości dokonanej estymacji punktowej. W artykule przedstawiono rozwiązania dwóch problemów badawczych: poszukiwania takiego estymatora bayesowskiego, który ma mniejszy błąd średniokwadratowy w porównaniu z ujęciem klasycznym dla z góry określonego przedziału, oraz przejrzystego sposobu modelowania rozkładów *a priori*.

Dzięki zidentyfikowaniu pewnych powiązań pomiędzy zmiennymi opisywanymi mieszankami rozkładów z rodziny Poisson-gamma możliwe stało się rozwiązanie obu wyżej sformułowanych problemów oraz zbudowanie prostego algorytmu optymalnej estymacji punktowej parametru rozkładu Poissona. Algorytm ten został wykorzystany do oszacowania liczby nowo tworzonych przedsiębiorstw.

Słowa kluczowe: bayesowska estymacja punktowa, błąd średniokwadratowy, rozkłady sprzężone, rozkład *a priori*, rozkład *a posteriori*, liczba tworzonych przedsiębiorstw.