

Sławomir Śmiech

Department of Statistics
Cracow University of Economics

Co-movement of Commodity Prices – Results from Dynamic Time Warping Classification

Abstract

Several factors are responsible for difficulties in describing the behaviour of commodity prices. Firstly, there are numerous different categories of commodities. Secondly, some categories overlap with other categories, while others indirectly compete in the market. Thirdly, although essentially commodity prices react to changes in economic conditions or exchange rates, to a large extent these prices depend on supply disturbances. However, in recent years commodity prices co-move, and researchers, beginning with Pindyck and Rotemberg (1990), have been trying to explain this phenomenon.

The objective of the article is to conduct the classification of the series of commodity prices in the pre-crisis and after-crisis periods. The results of such classification will reveal whether co-movement of commodity prices is the same in both periods. The analysis is based on monthly data from the period January 2001 to February 2014. All prices and price indices are published by the World Bank. The results obtained in dynamic time warping clustering reveal that co-movement of commodity prices is more evident in the pre-crisis period. There are only several paths which determine commodity prices.

Keywords: commodity prices, time series clustering, co-movement, dynamic time warping.

1. Introduction

Several factors are responsible for difficulties in describing the behaviour of commodity prices. Firstly, there are many different categories of commodities. Secondly, some categories overlap with others (for example, biofuel production and energy), while others indirectly compete in the market (for example, the development of one type of crop reduces the supply of other crops cultivated in a given area). Thirdly, although essentially commodity prices react to changes in economic conditions or exchange rates, to a large extent they depend on supply disturbances (such as droughts, floods, armed conflicts, etc.). In spite of the complex nature of commodity behaviour, the last decade saw them tend to move together. J. A. Frankel (2008) argues that the reason for such co-movement is the real interest rate, while F. Q. Akram (2009) additionally investigates the role of the dollar exchange rates, and L. E. O. Svensson (2008) and L. Kilian (2008, 2009) discuss the role of shifts in the global supply and demand. P. Krugman (2008) attributes the increase in food prices to biofuel production, as biofuel prices are correlated with oil prices. Numerous authors (e.g. Gilbert 2009, Phillips & Yu 2010, Irwin & Sanders 2011, Pindyck & Rotemberg 1990) reason that co-movement is caused by speculation and the existence of price bubbles. From a methodological point of view, several methods can be used to assess price co-movement. One of the most common is cointegration (Papież & Śmiech 2011), which has more recently been replaced by the panel cointegration approach (Nazlioglu & Soytas 2012), threshold cointegration (Natanelov *et al.* 2011) and the general equilibrium model (see e.g. Gohin & Chantret 2010). Other methods incorporate different statistical factor models, e.g. FAVAR and PANIC (Byrne, Fazio & Fiess 2013).

The objective of this paper is to classify a series of commodity prices using monthly data from three periods: before the global financial crisis (January 2001 to June 2008), after the crisis (January 2009 to February 2014), and the period covering the entire sample (January 2001 to February 2014). The prices of 54 commodities taken into consideration in the analysis are listed by the World Bank in six categories, i.e. energy, metals, beverages, food, raw materials and precious metals. Clustering was done with dynamic time warping methods, which enable the assessment of similarity between series shapes, i.e. a distance measure which identifies time-shifted patterns among series and seems to be appropriate for the analysis of commodity co-movement. Three methods are used to classify the time series: Ward's method, complete (hierarchical), and pam (division). The results of the classification are assessed by internal classification measuring the average silhouette width. The clustering conducted provides answers to the following questions:

- did commodity prices move at a similar rate in the periods before and after the global financial crisis?
- how many clusters of commodity prices are there and how homogeneous are they?
- do commodities from the same category (e.g. energy commodities) belong to the same clusters – that is, do their prices behave in a similar manner?
- to what extent do the clusters obtained in the study differ from the indices listed by the World Bank?

Our work differs from that found in the existing literature in one important respect – the methodology we use. Related studies have all assumed linear correlations. The methodology (i.e. dynamic time warping) used in this study, however, allows us to stretch or compress parts of two time series in order to draw comparisons.

Unlike traditional methods used to measure time series similarity, the high congruity of both paths does not necessarily mean that changes in trends occur at the same moment. Dynamic time warping allows us to delay changes in the trend of one commodity price series in relation to the other, which may be caused by different levels of inventories, among other things. This is why the methodology can be used as a universal analysis of the nonlinear relationship and co-movement of commodity prices.

The rest of the paper is organised as follows. Section 2 describes the methodology, section 3 discusses the empirical results, and the final section provides a discussion and conclusion.

2. Methodology

Following the division suggested by W. T. Liao (2005), there are three major time series clustering approaches: raw data, feature-based and model-based. The first ones deal with raw data in the time and frequency domain. They imply working with high dimensional space and are not effective if the raw data are highly noisy. In feature-based approaches, certain features are extracted then clustered. Y. Kakizawa, R. H. Shumway and M. Taniguchi (1998) characterise similarities of multivariate stationary time series in terms of their covariance or, equivalently, spectral metrics. Model-based approaches assume that each time series is generated by a particular time series model. To obtain dissimilarity between series, the models are fitted and then discrepancies between them are looked for. Some authors suggest using some statistics of the errors associated with the estimates (Kumar & Patel 2007). The main disadvantage of the feature-based and model-based approaches is the obvious loss of information. What is more, the

results of clustering in these methods depend on feature selection and problems with parametric modelling. A. Alonso *et al.* (2006) suggest another classification approach – n clustering based on the models that generated the observations, but in respect of the forecasts at a specific future time.

One of the most widely used methods of assessing similarity in the raw data approach is Dynamic Time Warping (DTW) (Berndt & Clifford 1994). Given two time series, $Q = q_1, q_2, \dots, q_n$ and $R = r_1, r_2, \dots, r_m$, DTW aligns them in such a way as to minimise their difference. The metric establishes an n by m cost matrix C , which contains the distances (Euclidean) between two points q_i and r_j . A warping path $W = w_1, w_2, \dots, w_K$, where $\max(m, n) < K < m + n - 1$, is formed by a set of matrix components, respecting three rules: boundary condition, monotonicity condition and step size condition. Eventually, the path that minimises the warping cost is considered the DTW distance:

$$d_W(R, Q) = \min \left(\sqrt{\sum_{k=1}^K w_k} \right). \quad (1)$$

The optimal warping path can be found using dynamic programming to evaluate the following expression:

$$\gamma(i, j) = d(r_i, q_j) + \min \{ \gamma(i-1, j-1), \gamma(i-1, j), \gamma(i, j-1) \}, \quad (2)$$

where:

$d(r_i, q_j)$ – the distance found in the current cell,

$\gamma(i, j)$ – the cumulative distance of $d(r_i, q_j)$ and the minimum cumulative distances from the three nearby cells.

After determining the distance matrices, hierarchical or partitioning (crisp or fuzzy) clustering methods are used to find clusters. Silhouette plot width (Kaufman & Rousseeuw 1990) is used to evaluate the optimal number of clusters in the data and to assess which objects lie well within their cluster internal validity indices. The silhouette width is defined for each sample and takes values from -1 to 1 . If the value is close to 1 , the object is near the centre of the cluster it belongs to. Conversely, if the value is negative, the object is in an improper cluster. Finally, if the silhouette value is close to null, the sample is located near the frontier between its cluster and the nearest one. The higher the value of average silhouette width, the better the division of the series.

Adjusted Rand Index (ARI) can be next applied to compare the alternative classification results. The Adjusted Rand Index was created by L. Hubert and P. Arabie (1985), who used the generalised hypergeometric distribution as the model of randomness. The index has an expected value of zero (for independent clustering) and a maximum value of 1 (for identical clustering). The higher the Adjusted Rand Index, the greater the agreement between the clustering results will be.

3. Data and Empirical Results

The data used in this study consist of monthly price indices from January 2001 to February 2014. All indices came from World Bank Commodity Price Data and are expressed in US dollars. Before the classification procedure, all price series are expressed as indices with their average values in 2007 set to be 1. The analysis is based on 54 series of variables, which are assigned to World Bank classes. The whole sample period is divided into two sub-periods: January 2001–June 2008 and January 2009–February 2014, thus the classification is based on the pre-crisis and post-crisis periods. The results are complemented by clustering series in the whole sample. The division is motivated by the disparate behaviour of commodity prices in these sub-periods. DTW methods are used to classify the time series. After obtaining dissimilarity metrics, Ward's, complete (hierarchical group of methods) and pam (division) methods are used to locate the clusters.

The results of clustering for the period 2001–2008 are presented in Fig. 1¹. They yield three main clusters of time series (the average silhouette width is the biggest for three clusters in Ward's method – see Table 1²). The first cluster consists of 28 commodity prices including most energy commodities (those in Fig. 1 beginning with E), except for Gas US, metals (MM) besides aluminium, and precious metals (PM). What is more, commodities belonging to the same category are close to each other, meaning their series paths are quite similar. The prices from the categories listed above are closest to one another, so their paths are the most similar. The second cluster includes the prices of Gas US and Sugar EU, between which it is difficult to spot any connections. The last cluster consists of 24 commodity prices including most food, raw materials and beverages commodities. The silhouette plot indicates that most commodity prices have been assigned to proper clusters (Fig. 4). The silhouette width is negative in only three cases, indicating that objects have been classified in improper clusters. The average silhouette width for the first sub-period is 0.43.

The results of clustering for the post-crisis period, with the assumption of Ward's method, are presented in Fig. 2. Although in this case the average silhouette width suggests a division into 2 clusters, we have opted for three clusters, so energy, metal, and precious metals commodities are in different groups. There are 14 commodity prices in the first cluster, including oil prices (except for WTI

¹ The results of the complete method and pam method are available from the author upon request.

² Presentation of the Ward's method results is not dictated by the fact this method clearly gave the best clusters. In fact, none of the methods dominated the others. Even though the results of Ward's method was neither better nor worse as compared to other options, that method served well as an illustration of the issues discussed.

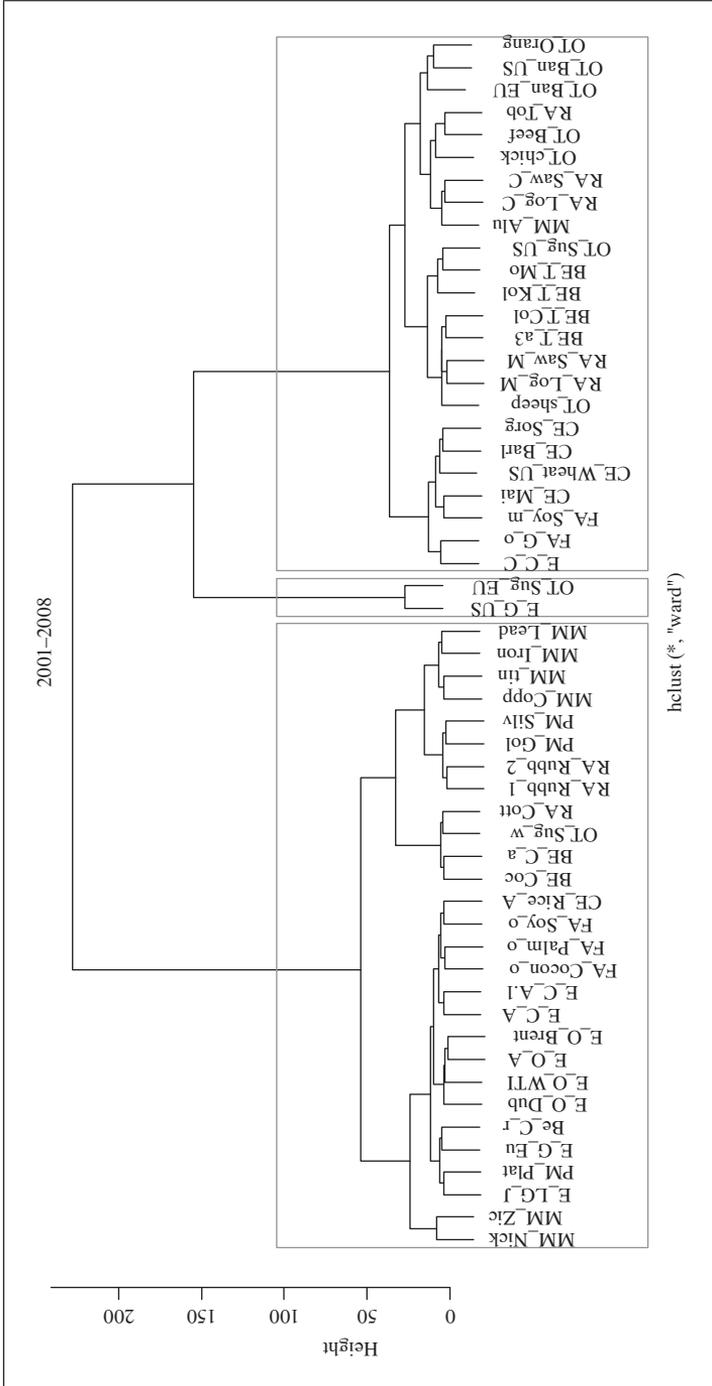


Fig. 1. The Classification Results in the First Sub-period
 Source: own calculations.

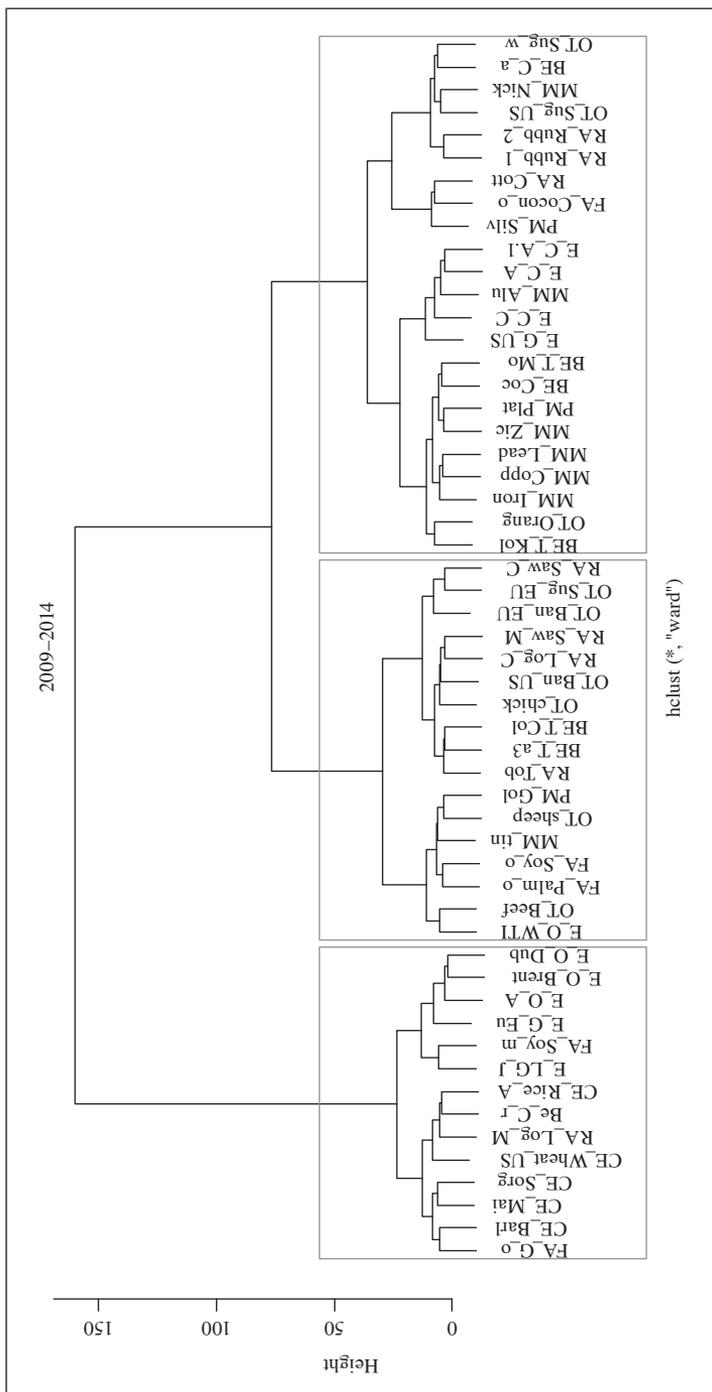


Fig. 2. The Classification Results in the Second Sub-period

Source: own calculations.

Oil, which is in the second cluster), and some food and raw material commodity prices. There are 19 elements in the second cluster, including most food and raw material prices, gold, and tin. There are 23 prices in the third cluster, including most precious metals, metals and minerals, coal prices and the remaining food commodities. The silhouette plot (Fig. 4) reveals that in the post-crisis period each cluster is less homogeneous than it was before. The average silhouette for the clusters varies between 0.17 and 0.35. There are five commodity prices that seem to have been classified in the wrong clusters. The average silhouette width calculated for the post-crisis period is only 0.26, suggesting that the structure obtained is artificial.

Finally, Fig. 3 presents the results obtained for the whole sample. Here the average silhouette width suggests (see Table 1) division into 4 groups (although the quality of the division is rather poor). The first cluster contains 17 elements including agricultural commodities (beverages, raw materials and other) and one industrial metal – aluminium. Most industrial and precious metals, Australian Coal and some other agricultural commodities are located in the second cluster, which contains 18 elements. The last two clusters are quite close to each other. The third consists of US Gas and Sugar UE, while the fourth contains most energy commodities and some food, especially oils (palms, soya, groundnut).

Table 1. The Average Silhouette Width for 2, 3 and 4 Clusters in Different Time Periods

Methods period\ nr cluster	Ward's			Complete			Pam		
	2	3	4	2	3	4	2	3	4
2001–2014	0.235	0.287	0.322	0.605	0.265	0.322	0.274	0.196	0.225
2001–2008	0.374	0.427	0.301	0.746	0.423	0.274	0.382	0.429	0.301
2008–2014	0.371	0.257	0.221	0.378	0.237	0.221	0.338	0.245	0.214

Source: own calculations.

In order to compare the results of classifications, the adjusted rand index is computed (see Table 2). The level of agreement of different classifications and the comparison of clusters and categories of different commodities (listed in the World Bank indices – symbol WB in Table 2) are measured. As there are six different commodity categories, the assumed division of the set of objects also consists of six clusters.

The results obtained reveal that commodity classifications do not determine similar behaviour of commodity prices, which is clearly seen in the low ARI values for the first and the second sub-periods (where the values are the highest) as well as for the whole sample period. As far as various methods of obtaining

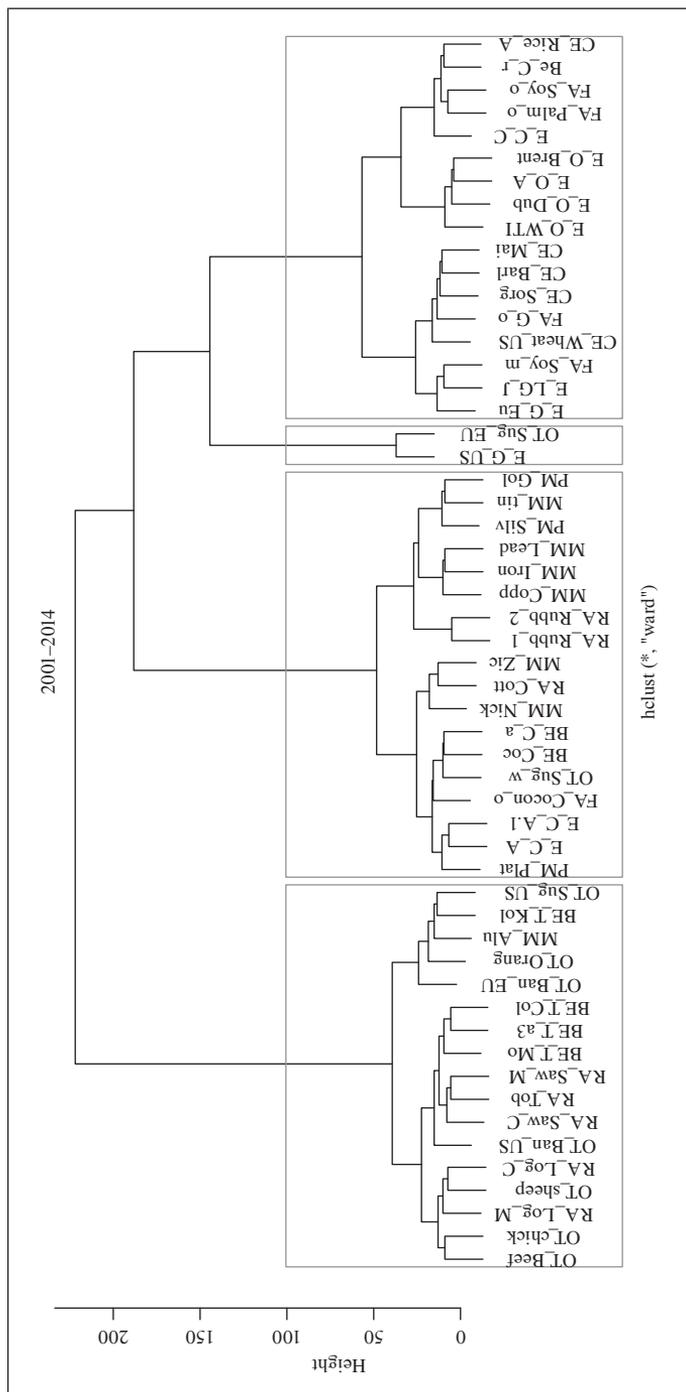


Fig. 3. The Classification Results for the Whole Sample Period

Source: own calculations.

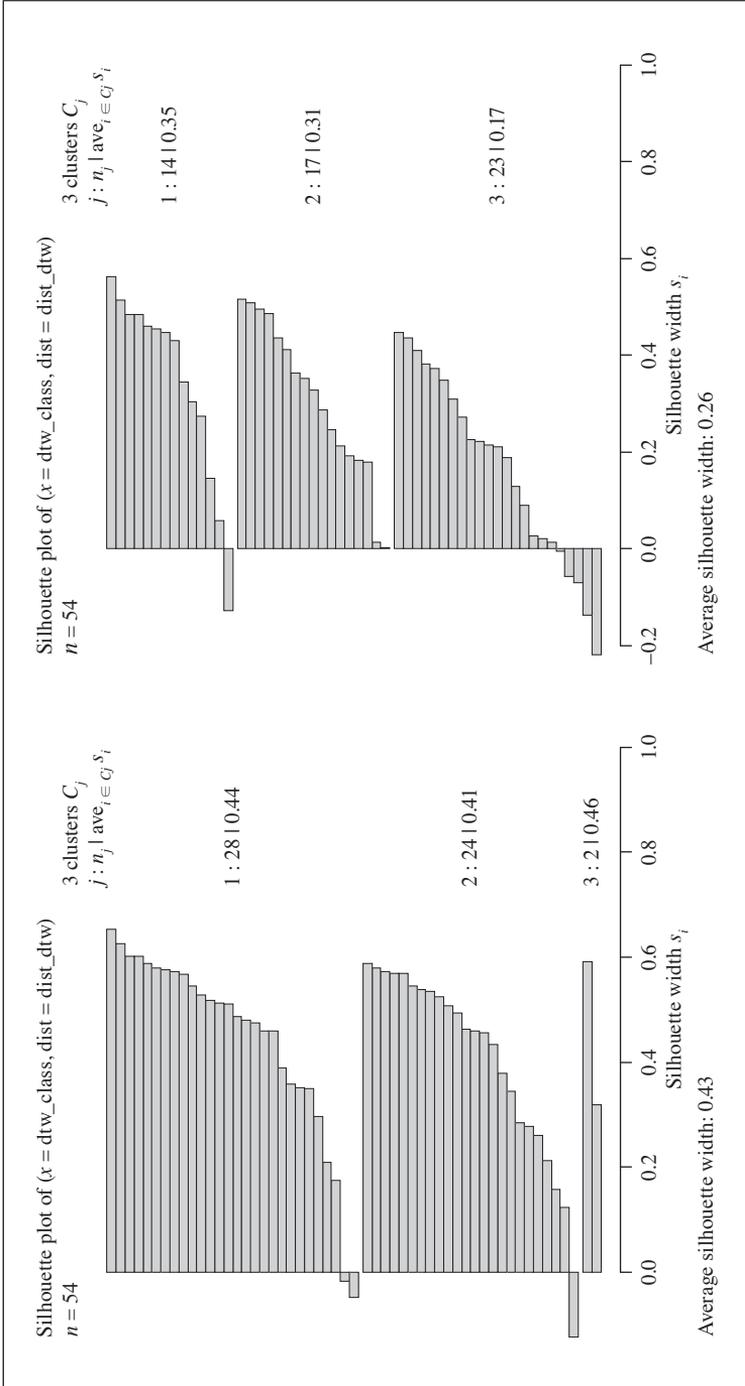


Fig. 4. Silhouette Plots for Pre-crisis (left panel) and Post-crisis (right panel) Period

Source: own calculations.

clusters are concerned, they are relatively high (from 0.467 obtained for pair *complete-ward* in second sub-period to pair *pam-complete* in the first sub-period). Again, higher values of the similarity measure are obtained in the first sub-period, which indicates that in this sub-period co-movement of the indexes is more evident, and is easily detected by different time series classification tools.

Table 2. Adjusted Rand Index for Different Classification Methods

Period	2001–2014			2001–2008			2009–2014		
	WB	Ward	compl.	WB	Ward	compl.	WB	Ward	compl.
WB	1			1			1		
Ward	0.100	1		0.139	1		0.077	1	
compl.	0.123	0.543	1	0.163	0.626	1	0.074	0.467	1
pam	0.167	0.415	0.496	0.125	0.588	0.688	0.114	0.490	0.569

Source: own calculations.

Table 3. Adjusted Rand Index for the Ward Results and Different Periods

Periods	Number of clusters			
	3 clusters		6 clusters	
	2001–2014	2001–2008	2001–2014	2001–2008
2001–2008	0.419	–	0.730	–
2009–2014	0.370	0.064	0.238	0.156

Source: own calculations.

In order to compare the composition of clusters in different periods, the ARI index is computed for 3 and 6 clusters. As Table 3 shows, the results obtained reveal that the composition of clusters in the pre-crisis and post-crisis periods differ greatly (in the division into 3 clusters ARI comes out to 0.064, while into 6 clusters it is 0.156). Relatively strong similarity of cluster composition in the pre-crisis sub-period and the whole period (ARI from 0.419 to 0.73 for 6 clusters) is attributable to the co-movement of all commodity prices in the pre-crisis period being stronger and more evident.

4. Conclusion and Discussion

Dynamic time warping has been used in the study to classify commodity price data in the pre-crisis and post-crisis periods. The results obtained reveal that co-movement of commodity prices is more evident in the pre-crisis period

when the clusters are more homogeneous and consist of commodities from the same category (e.g. precious metals or energy commodities are located in the same cluster). Clusters obtained for the post-crisis period are less homogeneous. The internal classification measure demonstrates that the best division is obtained if only two or three clusters are considered in every period. Clusters obtained for the whole period sample indicate that there are only two patterns of behaviour of prices in the periods analysed (stronger in the first one). Comparing commodity categories with the results of clustering indicates that commodities which belong to a single category do not always behave in the same way. This is especially evident in the second period, when certain energy commodities, metals or precious metals belong to different clusters. The results obtained might be of great importance to investors, as they demonstrate that the co-movement of commodity prices is not currently as evident as it was in the pre-crisis period. Consequently, the correlation between financial instruments is not high, and it is in general much easier to construct low variance portfolios. Moreover, because instruments belonging to the same class do not co-move (e.g. energy resources, metals or precious metals) it is possible to construct a well-diversified portfolio within the class.

To conclude, co-movement of commodity prices has not recently been as evident as it was in the pre-crisis period. What might be the reason for such change in investor behaviour? It may well be due to the disappearance of its causes, which include, according to popular explanations, low interest rates and inflation expectations, shifts in global supply and demand, and the risks resulting from geopolitical uncertainties and speculative bubbles. The first two seem still valid.

In the post-crisis period, real interest rates decreased. The crisis at first caused a dramatic demand slump, which gradually came back to its pre-crisis level. Due to difficulties with direct measuring, it is harder to refer to the remaining two causes of co-movement. It seems probable that the global financial crisis has led to increasing geopolitical risks, so it is justified to assume that co-movement has been caused by speculation. Thus, it is most likely that the crisis has changed investors' behaviour in the long run.

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Podobieństwo ścieżek cen towarów na rynkach światowych – analiza na podstawie klasyfikacji szeregów czasowych za pomocą metody dynamic time warping

(Streszczenie)

Wiele czynników powoduje problemy dotyczące modelowania zachowania cen towarów na rynkach światowych. Wśród nich wymienić można różnorodność kategorii towarów – które dodatkowo nie są rozłączne, powiązania cen towarów z różnych kategorii – część towarów może być traktowana jako komplementarne, przyczyny wahań cen towarów. Pewne przyczyny (głównie po stronie popytowej), takie jak aktywność gospodarcza, stopy procentowe czy kursy walut, są wspólne dla wielu kategorii towarów, inne z kolei – związane z podażą – są specyficzne. Mimo to w ostatnich dziesięcioleciach ceny towarów zachowują się podobnie (*co-move*), co doczekało się wielu opracowań.

W pracy przeprowadzono i oceniono grupowanie cen towarów w okresie przed kryzysem oraz po globalnym kryzysie finansowym w celu sprawdzenia, czy ceny towarów przed kryzysem i po kryzysie grupują się w podobne skupiska i czy homogeniczność tych skupisk jest podobna. Analiza została przeprowadzona na danych miesięcznych z okresu styczeń 2001–luty 2014. Wszystkie ceny oraz indeksy cen zaczerpnięto z bazy Banku Światowego. W badaniu wykorzystano metodę *dynamic time warping*, dzięki której wykazano, że wspólne zachowanie cen było silniejsze w okresie przed globalnym kryzysem finansowym. Ustalono także, że liczba skupisk jest niewielka, co oznacza, że można zauważyć tylko kilka tendencji w zakresie zachowania się cen na światowych rynkach towarowych.

Słowa kluczowe: ceny towarów, klasyfikacja szeregów czasowych, *dynamic time warping*, wspólne zachowanie cen.