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Changing Production on the Market with Continuum Traders

Abstract

Assume that in a Debreu private ownership economy, all consumption sets are contained in a proper subspace of the commodity-price space. This property motivates producers to change their activities on markets to plans from the same subspace. Alterations in production take time, so to model changes occurring in the production sector of an economy, time should be involved. In contrast to the results I obtained from a 2010 study, in the economy with continuum traders, excluding a finite number of producers from the process that adjusts the economy to the given requirements does not disturb equilibrium. The aim of this paper is to present the possible trajectories of producers' activity changing their plans in the economy with continuum traders. The process of moving the production system and, consequently, the whole economy to an economy reduced to a given subspace will be elaborated. As a result, a system of economies in equilibrium dependent on time is presented.

Keywords: economy with continuum traders, reduced consumption sphere, continuous trajectory of changes, projections.

1. Introduction

Since G. Debreu presented his results on the (static) private ownership economy (see e.g. Debreu 1959, 1982), many dynamic economic structures on the basis of the Debreu economy have been defined. Interesting results can be found

in (Radner 1970, Magill & Quinzii 2002). In (Lipieta 2010) and (Lipieta 2012) the evolution of the Debreu economy was also studied but by the assumption that the consumption system was contained in a proper subspace of the commodity-price space. As a result, the producers, tending to maximise their profits, made the decision to adjust the quantities of commodities in their production plans to a given relationship. In order not to destroy the existed equilibrium, all producers had to follow the same procedure and the fixed trajectory of changes. Without special supervision and leadership, these conditions are difficult to realise in real economies.

This paper is an attempt to adapt the results obtained in (Lipieta 2010) to an economy with continuum traders. The motivation of this research task was as follows: in an economy with continuum traders, the non-participation of a finite number (even a very large) of producers in the adjustment process earlier presented did not disturb equilibrium, if it existed.

The paper consists of four parts. The next section presents the construction of the private ownership economy with continuum traders. The third part describes the economies with a reduced production or consumption sphere. The fourth and final part focuses on the structure of action to be taken in the economy with dynamic production.

2. Model

I shall begin by presenting a set of definitions that will be of use later. The linear space $\mathbb{R}^{\ell}, \ell \in \{1, 2, \dots\}$, with the scalar product:

$$(x \circ y) = (x_1, x_2, \dots, x_{\ell}) \circ (y_1, y_2, \dots, y_{\ell}) = \sum_{k=1}^{\ell} x_k \cdot y_k$$

is interpreted as the ℓ – dimensional space of commodities and prices. The mapping:

$$proj_k \colon \mathbb{R}^{\ell} \ni (x_1, \dots, x_{\ell}) \to x_k \in \mathbb{R}, \qquad (2.1)$$

$k \in \{1, \dots, \ell\}$, is called the natural projection on the k-th coordinate.

Recall that if functions $f_1, \dots, f_k \colon D \to \mathbb{R}$ are Lebesque integrable on D, where $D \subset \mathbb{R}$ is a Lebesque measurable set, then function $f \stackrel{\text{def}}{=} (f_1, \dots, f_k) \colon D \to \mathbb{R}^k$ of the form:

$$f(x) = (f_1(x), \dots, f_k(x)) \text{ for } x \in D, \qquad (2.2)$$

is Lebesque integrable (compare to Aumann 1962). The integral of f, by (2.2), in the Lebesque sense, is of the form:

$$\int_D (x) d\mathcal{L}^1 \stackrel{\text{def}}{=} \left(\int_D f_1(x) d\mathcal{L}^1, \dots, \int_D f_k(x) d\mathcal{L}^1 \right).$$

The concept of the economy with continuum traders was introduced by R. Aumann. The basic assumptions and definitions of that model are presented below. According to Aumann's idea (see Aumann 1962), it is assumed that continuum of traders operate in $\mathbb{R}^{\ell}$. We assume additionally that both the number of consumers and the number of producers are continuum. The set of consumers is interpreted as closed interval [0, 1], which it is denoted by A. The sets of producers, denoted by B, is assumed to be the Lebesque measurable subset of interval [0, 1]. Inclusion $B \subset A$ means that every producer is also a consumer, but not every consumer has to be a producer. The producers tend to maximise their profits and the consumers want to maximise their utilities on the budget sets. If there exists a price vector $p \in \mathbb{R}^{\ell}$ such that the agents manage to realise their tasks at price system p and the market clearing condition is satisfied, then such vector p is called the equilibrium price vector.

Feasible production plans $y^b \in \mathbb{R}^{\ell}$, of producer b, form the production set Y^b of producer b. The correspondence:

$$\delta: B \ni b \to Y^b \subset \mathbb{R}^{\ell},$$

which to every producer b assigns production set $Y^b \subset \mathbb{R}^{\ell}$ of producer b is called the correspondence of production sets. We assume additionally (see Aumann 1962) that for every $s \in \{1, \dots, \ell\}$, each function of the form:

$$y_s: B \ni b \to y_s^b \in proj_s(Y^b) \subset \mathbb{R}, \tag{2.3}$$

where $y^b = (y_1^b, \dots, y_{\ell}^b) \in \mathbb{R}^{\ell}$ is a production plan of producer $b(y^b \in Y^b)$, is Lebesque integrable on B.

Analogously to (Lipieta 2010), the following may be posited:

Definition 2.1. If for the given price vector $p \in \mathbb{R}^{\ell}$:

$$\forall b \in B: \eta^b(p) \overset{\text{def}}{=} \{y^{b*} \in Y^b : p \circ y^{b*} = \max\{p \circ y^b : y^b \in Y^b\}\} \neq \varnothing,$$

then:

– correspondence $\eta: B \ni b \to \eta^b(p) \subset Y^b$ which to every producer b assigns the set $\eta^b(p)$ of production plans maximising his profit at price system p, is called the correspondence of supply at price system p,

– function $\neq: B \ni b \to \neq^b(p) = p \circ y^{b*} \in \mathbb{R}$ is the maximal profit function at price system p, where $y^{b*} \in \eta^b(p)$ for every $b \in B$.

Note that each function of the form:

$$B \ni b \to y^{b*} \in \eta^b(p),$$

for given $p \in \mathbb{R}^{\ell}$ is Lebesque integrable on B (by the assumption of integrability of the functions of the form (2.1)). The vectors from set $\eta^b(p)$ will be called the optimal plans of producer b.

Definition 2.2. The two-range relational system:

$$P = (B, \mathbb{R}^\ell; \delta, p, \eta, \pi)$$

is called the production system.

Similarly, let:

– $\chi\colon A \ni a \to X^a \subset \mathbb{R}^\ell_+$ be a correspondence of consumption sets which to every consumer $a \in A$ assigns the consumption set X^a representing the consumer's feasible consumption plans with respect to his psycho-physical structure; it is assumed that for every $s \in \{1, \ldots, \ell\}$, and $x^a \in X^a$ each function of the form:

$$x_s\colon A \ni a \to x_s^a \in proj_s(X^a) \subset \mathbb{R} \tag{2.4}$$

is Lebesque integrable on A.

– $\Xi \subset \mathbb{R}^\ell \times \mathbb{R}^\ell$ be the family of all preference relations defined on the commodity space $\mathbb{R}^\ell$;

– $e\colon A \ni a \to e^a \in X^a$ be an initial endowment mapping, which to every consumer $a \in A$ assigns his initial endowment vector $e(a) \overset{\text{def}}{=} e^a \in X^a \subset \mathbb{R}^\ell$;

– $\varepsilon\colon A \ni a \to \varepsilon(a) \subset X^a \times X^a$ be a correspondence, which to every consumer $a \in A$ assigns $\varepsilon(a) \overset{\text{def}}{=}$ a preference relation $\preccurlyeq^a$ from the set Ξ, restricted to the consumption set X^a;

– $p \in \mathbb{R}^\ell$ be a price vector.

Note that the expenditures of every consumer $a \in A$ cannot be greater than the value:

$$w^a = p \circ e(a). \tag{2.5}$$

The following definitions may be assumed on the basis of the above:

Definition 2.3. If for every $a \in A$ at given price vector $p \in \mathbb{R}^\ell$,

$$\beta^a(p, w^a) \overset{\text{def}}{=} \{x \in \chi(a)\colon p \circ x \le w^a\} \ne \varnothing$$

$$\xi^a(p, w^a) \overset{\text{def}}{=} \{x^{a*} \in \beta(p, w^a) \mid \forall x^a \in \beta^a(p, w^a)\colon x^a \preccurlyeq^a x^{a*}, \preccurlyeq^a \in \Xi\} \ne \varnothing, \tag{2.6}$$

then:

– $\beta\colon A \ni a \to \beta^a(p, w^a) \subset \mathbb{R}^\ell$ is the correspondence of budget sets at given price system p, which to every consumer $a \in A$ assigns his set of budget constrains $\beta^a(p, w^a) \subset \chi(a)$ with the price system p and the initial endowment $e(a)$;

– $\xi\colon A \ni a \to \xi^a(p, w^a) \subset \mathbb{R}^\ell$ is the demand correspondence at given price system p, which to every consumer $a \in A$ assigns his consumption plans maximising his preference on the budget set $\beta^a(p, w^a)$.

The vectors from set $\xi^a(p, w^a)$ will be called the optimal plans of consumer a.

Definition 2.4. The three-range relational system:

$$C = (A, \mathbb{R}^{\ell}, \Xi; \chi, e, \varepsilon, p, \beta, \xi)$$

is called the consumption system.

Let $p \in \mathbb{R}^{\ell}$ be a price vector, P – be a production system and C – a consumption system in the same space $\mathbb{R}^{\ell}$. Let us assume that some consumers share in the producers' profits. If $\theta(a, b)$ indicates that part of the profit of producer b which is owned by consumer a, then $0 \le \theta(a, b) \le 1$. Moreover, the mapping $\theta: A \times B \to [0, 1]$ satisfies:

$$\forall b \in B: \sum_{a \in A} \theta(a, b) = 1 \qquad (2.7)$$

and

$$\forall a \in A: \sum_{b \in B} \theta(a, b) \in \mathbb{R}.$$

In this situation the value w^a, $(a \in A)$ in (2.5) is changed by the rule:

$$w^a = p \circ e(a) + \sum_{b \in B} \theta(a, b) \pi^b(p). \qquad (2.8)$$

Let

$$\omega = \int_A e(a) d\mathcal{L}^1 \in \mathbb{R}^{\ell} \qquad (2.9)$$

be the total endowment in economy E_p (the initial endowment mapping e, by assumption (2.4) is Lebesque integrable on A). If for every $a \in A$ and w^a, given by (2.8), set $\xi^a(p, w^a) \ne \emptyset$ (see (2.6)), then the following definition is formulated:

Definition 2.5. The relational system:

$$E_p = (P, C, \theta, \omega)$$

is called the private ownership economy with continuum of traders.

The economy E_p operates as follows. Let a price vector $p \in \mathbb{R}^{\ell}$ be given. Every producer $b \in B$ chooses a production plan $y^{b*} \in \eta^b(p)$ maximising his profit at price system p. Hence, the mapping:

$$y: B \ni b \to y^{b*} \in \eta^b(p),$$

is specified. The vector:

$$y^* = \int_B y(b) d\mathcal{L}^1 \qquad (2.10)$$

is called the equilibrium total production plan. The maximal profit of each producer is divided among all consumers according to function θ. Now the expenditures of every consumer $a \in A$ cannot be greater than value w^a calculated by (2.8). In this situation, every consumer a chooses his consumptions plan

$x^{a*} \in \xi^a(p, w^a) \subset X^a$ maximising his preference on the budget set $\beta^a(p, w^a)$. By the above, the mapping:

$$x: A \ni a \to x^{a*} \in \xi^a(p, w^a) \subset X^a$$

is specified. The vector:

$$x* = \int_A x(a) d\mathcal{L}^1 \tag{2.11}$$

is called the equilibrium total consumption plan. If

$$x* - y* = \omega, \tag{2.12}$$

where ω is the total endowment in economy E_p (see (2.7)), then vector p is called the equilibrium price vector and is denoted by $p*$. Consequently, the set

$$(\{x^{a*}\}_{a \in A}, \{y^{b*}\}_{b \in B}, p*) \tag{2.13}$$

is called the state of Walras' equilibrium in economy E_p. It will be said that equilibrium exists in economy E_p, if condition (2.12) is satisfied for some set of form (2.13) with the total production plan of form (2.10) and the total consumption plan of form (2.11).

Remark 2.6. Let E_p be the private ownership economy with continuum traders in which condition (2.12) is satisfied and set (2.13) is the state of the Walras' equilibrium. Suppose that the producers from a set $\tilde{B} \subset B$ begin to realise other plans maximising their profits at given price system p. If $\tilde{B}$ is the set of measure Lebesque zero, then the equilibrium total production plan (see (2.10)) will remain unchanged. Consequently, condition (2.12) will be still satisfied in economy E_p. The same is applied to the consumers' plans.

3. Economy with a Reduced Consumption or Production Sphere

In this part of the paper, it is assumed that there exists a proper subspace $V \subset \mathbb{R}^\ell$ such that:

$$X^a \subset V \text{ for almost } a \in A. \tag{3.1}$$

Firstly, let us recall that it is said that the given property is satisfied "for almost $a \in A$" if there exists a subset $\tilde{A} \subset A$ of measure Lebesque zero, such that for every $a \in A \backslash \tilde{A}$ this property is true (set $\tilde{A}$ can be empty). Secondly, if V is a subspace of dimension $\ell - k$ ($k \in \{1, \ldots, \ell - 1\}$) of space $\mathbb{R}^\ell$, then there exist linearly independent vectors $h^1, \ldots, h^k \in \mathbb{R}^\ell$ ($h^s = (h_1^s, \ldots, h_\ell^s), s \in \{1, 2, \ldots, k\}$) satisfying:

$$V = \bigcap_{s=1}^k \ker \tilde{h}^s, \tag{3.2}$$

where:

$$\tilde{h}^s: \mathbb{R}^\ell \ni x \to h_1^s x_1 + \cdots + h_\ell^s x_\ell \in \mathbb{R} \qquad (3.3)$$

are, for every $s \in \{1, 2, \ldots, k\}$, the linear and continuous functions.

I will show that property (3.1) is often satisfied in real economies – specifically if at least one commodity is undesired by almost all consumers or if at least two commodities are complementary (see Lipieta 2010).

Precisely, if a commodity $j_0 \in \{1, \ldots, \ell\}$ is undesired by consumers, then for almost $a \in A$ and every consumption plan $x^a \in X^a$:

$$x_{j_0}^a = 0.$$

Hence, for almost $a \in A$, $X^a \subset V = \ker \tilde{h}$, where $\tilde{h}$ is of the form (3.3) with h defined by:

$$h = \begin{cases} 1 & \text{for } j = j_0 \\ 0 & \text{for } j \neq j_0 \end{cases}. \qquad (3.4)$$

In this case, $V = \ker \tilde{h} = \mathbb{R}^{\ell-1}$. Generally (if there are exactly $\ell - k$ different undesired commodities), $V = \mathbb{R}^k$ for some $k \in \{1, \ldots, l - 1\}$. If two commodities j_1, $j_2 \in \{1, \ldots, l\}, j_1 \neq j_2$ are complementary then condition (3.1) is also satisfied for almost all consumption sets. Then, there exists $c > 0$ such that:

$$V = \{x \in \mathbb{R}^\ell : x_{j_1} = c \cdot x_{j_2}\}.$$

Hence $V = \ker \tilde{h}$, where $\tilde{h}: \mathbb{R}^\ell \ni x \to x_{j_1} - c \cdot x_{j_2} \in \mathbb{R}$ is the linear and continuous functional and consequently $V \neq \mathbb{R}^k$ for any $k \in \{1, \ldots, \ell - 1\}$.
The economy E_p (see def. 2.5), in which condition (3.1) is satisfied for some subspace V of commodity-price space $\mathbb{R}^\ell$, will be called the economy with a reduced consumption sphere.

Note that production sets Y^b ($b \in B$) sometimes satisfy condition:

$$Y^b \subset V \text{ for almost } b \in B. \qquad (3.5)$$

Condition (3.5) can be satisfied, if the quantities of inputs or the quantities of outputs are proportional in production plans or if there is a commodity which for almost all producers is neither output nor input. Generally, production sets do not have to be contained in a subspace of commodity space $\mathbb{R}^\ell$. The economy E_p (see def. 2.5), in which condition (3.5) is satisfied for some subspace V of commodity-price space $\mathbb{R}^\ell$, will be called the economy with a reduced production sphere.

4. The Economy with a Dynamic Production System

The nonexistence of equilibrium in the real economies often leads to changes in the production sphere. Generally, if equilibrium exists, the producers do not

have incentives to change their production plans. However, in the spirit of perfect competition, the producers know about all properties of consumption plans. This knowledge, the desire to mimimise losses and, in consequence, to earn more money may cause producers to decide, even equilibrium in the economy exists, to change the production sphere. Every change of the production sphere may lead to a disturbance of the equilibrium in a given economy.

The procedure for changing the production sphere without disturbing the equilibrium was elaborated (see Lipieta 2010) on the example of a private ownership economy in which all consumption sets were contained in a proper subspace of the commodity-price space. However, the assumption of the finite number (often very large) of agents implied that if at least one producer did not follow the established trajectory of changes of the production systems, then the equilibrium would be destroyed. In the economy with continuum traders this problem does not exist. Condition (2.12) is satisfied, even some (but not too many, see Remark 2.6) producers will not follow the established adjustment procedure (the set of producers whose trajectories of the change differ from the others should be of the measure Lebesque zero). Below, the modifications of the production sphere which do not destroy the equilibrium at the given price system in the economy with continuum traders are presented.

Let $E_p = (P, C, \theta, \omega)$ be a private ownership economy with continuum traders (see def. 2.5), where condition (3.1) is satisfied. Hence the economy E_p is the economy with reduced consumption sphere. Suppose that subspace V by condition (3.1) is of dimension $\ell - k$ $(k \in \{1, \ldots, \ell - 1\})$ and of the form (3.2) with linearly independent functionals $\tilde{h}^1, \ldots, \tilde{h}^k$ satisfying (3.2). Let $q^1, \ldots, q^k \in \mathbb{R}^\ell$ be a solution of the system of equations:

$$\tilde{h}^s(q^r) = \delta^{sr} \text{ for } s, r \in \{1, \ldots, k\},\tag{4.1}$$

where:

$$\delta^{sr} = \begin{cases} 1 & \text{if } s = r \\ 0 & \text{if } s \neq r \end{cases}$$

is Kronecker delta. Let mapping $\hat{Q} : \mathbb{R}^\ell \times [0,1] \to \mathbb{R}^\ell$ be of the form:

$$\hat{Q}(x,t) = x - t \cdot \sum_{s=1}^{k} \tilde{h}^s(x) \cdot q^s.\tag{4.2}$$

We say that vectors $q^1, \ldots, q^k$ assign the direction of mapping $\hat{Q}$. For a given $t \in [0,1]$ and vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ satisfying (4.1) the mapping is defined as:

$$\hat{Q}_t : \mathbb{R}^\ell \to \mathbb{R}^\ell, \ \hat{Q}_t(x) \stackrel{\text{def}}{=} \hat{Q}(x,t).\tag{4.3}$$

As before, we say that vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ assign the direction of mapping $\hat{Q}_t$. Note that the mapping $\hat{Q}_t$ obtained by (4.2) with vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ satisfying

(4.1) is a unique one if and only if $k = \ell - 1$. If $k \in \{1, \ldots, \ell - 2\}$ then the system of equalities (4.1) has more than one solution.

The mapping $Q: \mathbb{R}^\ell \to \mathbb{R}^\ell$, $Q(x) \overset{\text{def}}{=} \hat{Q}_1(x) = \hat{Q}(x,1)$ satisfying

$$Q(x) = x - \sum_{s=1}^{k} \tilde{h}^s(x) \cdot q^s \qquad (4.4)$$

is the linear and continuous operator. Moreover

$$\forall x \in \mathbb{R}^\ell \, Q(x) \in V \text{ and } \forall v \in V \, Q(v) = v. \qquad (4.5)$$

Note by (4.5) that mapping Q is the linear and continuous projection from $\mathbb{R}^\ell$ into V (see Cheney 1966). We also say that vectors $q^1, \ldots, q^k$ assign the direction of projection Q.

Note that if $p \notin V^T$, where subspace $V \subset \mathbb{R}^\ell$ is of the form (3.2), then the system of equalities

$$\begin{cases} \tilde{h}^s(x) = \delta^{sr} \\ p \circ x = 0 \end{cases} \quad s, r \in \{1, \ldots, k\}, \qquad (4.6)$$

for every $r \in \{1, \ldots, k\}$ has a solution. We say that vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ satisfy assumption $\mathcal{A}$, if they satisfy condition (4.1) and additionally condition (4.6) if $p \notin V^T$.

Keeping the above assumptions and reasoning analogously as in the proof of theorem 4.2 in (Lipieta 2010), we determine the following to be true:

Theorem 4.1. Let E_p be an economy with continuum traders satisfying (3.1) with subspace $V \subset \mathbb{R}^\ell$. Let mapping $\hat{Q}: \mathbb{R}^\ell \times [0,1] \to \mathbb{R}^\ell$, of the form (4.2), be assigned by vectors $q^1, \ldots, q^k \in \mathbb{R}^l$ satisfying assumption $\mathcal{A}$. Then:

1) for every $b \in B$, if $y^{b*} \in \eta^b(p)$ then for every $t \in [0,1]$, vector $\hat{Q}(y^{b*}, t)$ maximises, at price p, the profit of producer b on the modified production set:

$$\hat{Q}(Y^b, t) = \{\hat{Q}(y^b, t) \in \mathbb{R}^\ell : y^b \in Y^b\};$$

2) for every $a \in A$, if $x^{a*} \in \xi^a(p, w^a)$, then x^{a*} maximises, at price p, the preference of consumer a on the set:

$$\left\{ x \in X^a : p \circ x \leq p \circ e^a + \sum_{b \in B} \theta(a,b) \cdot (p \circ \hat{Q}(y^{b*}, t)) \right\}.$$

Consider $t \in [0,1]$ and mapping $\hat{Q}_t$, of the form (4.3) assigned by vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$, satisfying assumption $\mathcal{A}$. Theorem 4.1 guarantees that the images of the optimal plans of every producer b by mapping $\hat{Q}_t$, maximise at the same price system, the profit of every producer b on the modified production set:

$$\hat{Q}_t(Y^b) = \{\hat{Q}(y^b, t) \in \mathbb{R}^\ell : y^b \in Y^b\}.$$

Let vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ satisfy assumption $\mathcal{A}$ and $t \in [0,1]$. We now introduce:

Definition 4.2. The two-range relational system:

$$P_t(q^1,\ldots,q^k)=(B,\,\mathbb{R}^\ell;\delta_t,\,p,\,\eta_t,\,\pi_t),\tag{4.7}$$

where:

– $\delta_t\colon B\ni b\to\hat{Q}_t(Y^b)\subset\mathbb{R}^\ell$ is the correspondence of production sets, which to every producer $b\in B$ assigns the image of production set Y^b by mapping $\hat{Q}_t$;

– $p=p^*\in\mathbb{R}^\ell$ is the price vector in the private ownership economy E_p;

– $\eta_t\colon B\ni b\to\eta_t^b(p)\subset\mathbb{R}^\ell$ is the correspondence of supply, which to every producer $b\in B$ assigns set $\eta_t^b(p)$ of production plans maximising his profit at the price system p on the set $\hat{Q}_t(Y^b)$, where:

$$\forall b\in B\;\eta_t^b(p)\overset{\text{def}}{=}\{\hat{Q}_t(y^{b*})\colon p\circ y^{b*}=\max\{p\circ y^b\colon y^b\in Y^b\}\}.$$

Note that:

$$\forall b\in B\;\eta_t^b(p)=\{\hat{Q}_t(y^{b*})\colon p\circ\hat{Q}_t(y^{b*})=\max\{p\circ\hat{Q}_t(y^b)\colon y^b\in Y^B\}\};$$

– $\neq_t\colon B\ni b\to p\circ\hat{Q}_t(y^{b*})\in\mathbb{R}$ is the maximal profit function and $y^{b*}\in\eta^b(p)$ for every $b\in B$,

is the evolution of production system $P=(B,\mathbb{R}^\ell;\delta,\,p,\,\eta,\,\pi)$ (by def. 2.2) at time t, on the trajectory assigned by vectors $q^1,\ldots,q^k$.

Note further that the relational system $P_t(q^1,\ldots,q^k)$ is the production system in the meaning of definition 2.2. Moreover, the production system $P_t(q^1,\ldots,q^k)$, besides price system p, is the image of production system P of economy E_p, by the mapping $\hat{Q}_t$ of the form (4.3). If $t=1$, then the production system P takes the final position, namely $P_1(q^1,\ldots,q^k)$, in which all production sets are contained in subspace V (as the images of production sets by initial production system P by projection $\hat{Q}_1$).

Consider vectors $q^1,\ldots,q^k\in\mathbb{R}^\ell$ satisfying assumption $\mathcal{A}$. Definition 4.2 directly leads to:

Definition 4.3. Let $t\in[0,1]$ and the mapping $\hat{Q}_t$ of form (4.6) in the direction of vectors $q^1,\ldots,q^k\in\mathbb{R}^\ell$ be given. The relational system:

$$E_t(q^1,\ldots,q^k)=(P_t(q^1,\ldots,q^k),C,\theta,\omega),\tag{4.8}$$

where:

– $P_t(q^1,\ldots,q^k)$ is given by (4.7),

– C is the consumption system contained by assumption (3.1), besides the price system, which may be, but is not necessarily, in subspace V,

will be called the evolution of the private ownership economy E_p at time t in the direction of vectors $q^1,\ldots,q^k\in\mathbb{R}^\ell$.

Note that the relational system $E_t(q^1,\ldots,q^k)$ is the private ownership economy with continuum of traders in the meaning of definition 2.5. As the result of the

evolution of the production system, assigned by vectors $q^1,\ldots,q^k \in \mathbb{R}^\ell$, we get the private ownership economy with continuum of traders $E_t(q^1, \ldots, q^k)$ with reduced consumption and production spheres.

Now we have:

Lemma 4.4. Let $t \in [0,1]$ and mapping $\hat{Q}_t$ of form (4.2) be assigned by vectors $q^1,\ldots,q^k \in \mathbb{R}^\ell$ satisfying assumption $\mathcal{A}$. If the set:

$$(\{x^{a*}\}_{a \in A}, \{y^{b*}\}_{b \in B}, p^*) \tag{4.9}$$

(see (2.13)) is the state of Walras equilibrium in economy E_p then set:

$$(\{x^{a*}\}_{a \in A}, \{\hat{Q}_t(y^{b*})\}_{b \in B}, p^*) \tag{4.10}$$

will be the state of Walras equilibrium in economy $E_t(q^1, \ldots, q^k)$.

Proof. The proof is the immediate consequence of theorem 4.1.

Suppose that private ownership economy with continuum traders E_p, satisfying condition (3.1) with subspace $V \subset \mathbb{R}^\ell$, is in equilibrium at price vector $p \in \mathbb{R}^\ell$. If vectors $q^1,\ldots,q^k \in \mathbb{R}^\ell$ satisfy assumption $\mathcal{A}$, then economy $E_t(q^1, \ldots, q^k)$, defined in (4.8), also has an equilibrium at price vector p. By given equilibrium price vector $p^* = p$, maximal profits of every producer b are constant in every economy $E_t(q^1, \ldots, q^k), t \in [0,1]$. This means that for every $b \in B$:

$$\forall t \in [0,1] \neq_t(b) = \neq(b).$$

Let $t \in [0,1]$ and vectors $q^1,\ldots,q^k \in \mathbb{R}^\ell$ satisfy assumption $\mathcal{A}$. The set of economies $\{E_t(q^1,\ldots,q^k), t \in [0,1]\}$ can be used to analytically describe the process of moving economy $E_p = E_0(q^1, \ldots, q^k)$ towards economy $E_1(q^1, \ldots, q^k)$. Theorem 4.1 guarantees the existence of equilibrium in every economy $E_t(q^1, \ldots, q^k), t \in [0,1]$, if it exists in initial economy $E_0(q^1, \ldots, q^k)$. This motion can be interpreted as the producers' reply for the dependency between the quantities of commodities in consumers' plans. On the other hand, it also leads to a simplification of the geometric structure of economy E_p because economy $E_1(q^1, \ldots, q^k)$ has reduced production and consumptions spheres. Further note that if $k = \ell - 1$, then the system of equations (4.1) has only one solution. Hence, then, there is only one trajectory of the changing production sphere, defined by mapping $\hat{Q}$ of the form (4.2), which does not destroy the equilibrium in every economy $E_t(q^1, \ldots, q^k)$, $t \in [0,1]$. If $k \in \{1,\ldots,\ell-2\}$ then the system of equalities (4.1) has more than one solution. This means that there are infinite paths (namely mappings $\hat{Q}$) to reach the final position $P_1(q^1, \ldots, q^k)$, which is the reduced form of production system and consequently the economy $E_1(q^1, \ldots, q^k)$ with reduced production and consumptions spheres.

I will now discuss the structure of the modification of the production sphere in private ownership economy E_p with continuum traders. The producers know that all consumption sets are contained in subspace $V \subset \mathbb{R}^\ell$. If producers realise the production plans, satisfying the dependency between quantities of commodities seen in consumption plans, then they could bring about the nonexistence of the surplus outputs. This reasoning compels the producers to adjust their activities to offer production plans that precisely fit consumers' requirements (meaning plans from subspace V). Every producer will change his production plans regardless of other producers, moreover each producer should conduct alterations according to a fixed recipe – the trajectory of changes, assigned by one of the mappings of the form (4.2). Hence, it seems that the procedure for changing the production system without destroying equilibrium in the economy requires action from an institution or person (below called the designer) guiding this process to achieve the desired goals. Firstly, the designer should establish when the adjustment process will begin and finish and which trajectory of the changing (precisely speaking: which vectors $q^1, \ldots, q^k \in \mathbb{R}^\ell$ satisfying assumption $\mathcal{A}$) to choose. Finally, production system $P_0(q^1, \ldots, q^k)$ will be changed to (production) system $P_1(q^1, \ldots, q^k)$ (see (4.7)) and the modification of the production system will be finished at (ending) point $t = 1$ common for all producers. During this process, economy $E_t(q^1, \ldots, q^k)$, interpreted as the state of economy $E_0(q^1, \ldots, q^k)$ at point t, remains in equilibrium at every point $t \in [0,1]$. If some producers from the subset of B, of measure Lebesque zero, do not follow the procedure, at arbitrary point t, then the economy remains in equilibrium at this point t.

The following scenario is also possible: almost all producers who decided to change their production plans chose the same trajectory of changes (mapping $\hat{Q}$ of the form (4.2)), because of the nature of the process or the nature of the (complementary or undesired) commodities. If even the producers start the procedure of changing not in the same point of time, then, after almost all of the producers have finished the adjustment process, the economy will gain the equilibrium. This follows from the form of mapping $\hat{Q}$ (see (4.2)). So, in this case, there is no designer and the producers' activity is caused by competitive mechanism.

5. Conclusion

The idea of an economy with continuum traders and a reduced consumption sphere makes it possible to model the evolution of the production sphere without losing equilibrium. Moreover, if "a small" number of producers do not take part in the process, the equilibrium will not be disturbed. This reflects the situation seen in real economies, where some producers do not follow the general tendency of

changes and the economy remains in equilibrium. It is not possible to model such a situation in a Debreu private ownership economy.

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Zmiana produkcji na rynku z nieskończoną liczbą agentów
(Streszczenie)

W pracy rozważono ekonomię Debreu z własnością prywatną, w której wszystkie zbiory konsumpcji są zawarte we właściwej podprzestrzeni przestrzeni towarów i cen. Ta własność motywuje producentów do zmiany uwidaczniającej się w realizacji planów z tej samej podprzestrzeni działalności na rynkach. Zmiany w procesie produkcji wymagają czasu, stąd w modelowaniu zmian w sektorze produkcji należy użyć zmiennej czasowej.

W odróżnieniu od wyników otrzymanych przez autorkę we wcześniejszych badaniach (2010), w ekonomii z nieskończoną liczbą agentów wykluczenie skończonej liczby producentów z procesów transformacyjnych nie zaburza równowagi. Celem artykułu jest prezentacja trajektorii zmian działalności producentów w ekonomii z nieskończoną liczbą agentów, w wyniku których otrzymujemy ekonomię zredukowaną do pewnej podprzestrzeni przestrzeni towarów i cen. Wynikiem jest układ ekonomii w równowadze zależnych od czasu.

Słowa kluczowe: ekonomia z nieskończoną liczbą agentów, zredukowany system konsumpcji, ciągła trajektoria zmian, projekcje.