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# On Some Construction of the Design of Experiments for Two Response Variables

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## ABSTRACT

**Objective:** The aim of the article is to propose an alternative approach to constructing a design of experiments that allows for identifying settings of factor levels so that the response variables achieve desired values. The presented method will be used to determine the parameters of the wood pellet production process, which is characterised by two response variables.

**Research Design & Methods:** This paper presents a proposal for the construction of an experimental design that allows for the inclusion of two outcome variables characterising the production process under study. The method employs an appropriate synthetic variable and considers the desired ranges of variation of the outcome variables. Furthermore, permutation tests were utilised in the analysis of the experimental results.

**Findings:** A method of constructing an experimental plan was proposed, which allowed unambiguous recommendations to be made for the wood pellet production process under study. The settings of the individual factors for which the quality of the pellets produced and the efficiency of the production line take on values within the specified range were indicated.

**Implications/Recommendations:** The presented method of constructing the experimental plan, based on the analysis of an appropriately constructed synthetic variable, allowed the wood pellet production process to be designed accordingly.

**Contribution:** An alternative method of constructing an experimental design has been proposed that allows for the analysis of the influence of factors on the two outcome variables that characterise the production process under study.

**Article type:** original article.

**Keywords:** design of experiments, response surface function, response variable, wood pellet production process.

**JEL Classification:** C99.

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## 1. Introduction

The advanced technological development contributes to the search for new solutions during the production process. In particular, modern manufacturing companies are seeking methods and tools to enhance the technological and economic results of the production process. Among the solutions that address these needs are statistical quality control methods, and in particular, methods of design of experiments (Kończak, 2007).

In the area of designing or improving manufacturing process outcomes, the use of experiment planning methodologies allows for tangible benefits, including a reduction in the time or cost of implementing a manufacturing process (Montgomery, 2020). Typically, experiment planning methods allow the evaluation of the dependence of a single response variable on a fixed number of factors. Contemporary manufacturing processes are complex, and many times the analysis of a single variable characterising the process under study will not be sufficient.

The objective of this article is to propose an alternative method of constructing an experimental plan that allows two response variables to be considered simultaneously in the experiment. The proposed method will be used to improve the results of a real wood pellet production process.

## 2. Basics of Design of Experiments Methods

In the 1920s, Ronald Aylmer Fisher was the first to propose the use of methods for the design of experiments in agricultural experimentation (Fisher, 1925, 1935). During the following decades, these methods were widely developed and found application not only in the natural sciences, but also in the practice of production enterprises. Nowadays, one sees opportunities for the use of design of experiments methods in the activities of non-manufacturing enterprises or in marketing (Antony *et al.*, 2011; Montgomery, 2020).

An experiment consists of  $n$  experimental trials. In the experiment, the values of the response variable  $Y$  are obtained with fixed values of the factors  $X_1, X_2, \dots, X_m$ .

The design of an experiment is the determination of an appropriate arrangement of the levels of the selected factors in the individual experiments, which should be carried out in a randomised manner. The dependence of the response variable  $Y$  on the values of the factors  $X_1, X_2, \dots, X_m$  is defined in the form of a statistical model (Wawrzynek, 1993):

$$Y(X_1, X_2, \dots, X_m) = y(X_1, X_2, \dots, X_m) + \varepsilon, \quad (1)$$

where  $EY(X_1, X_2, \dots, X_m) = y(X_1, X_2, \dots, X_m)$ ,  $E(\varepsilon) = 0$ ,  $V(\varepsilon) = \sigma^2$  and  $\sigma^2$  is constant. Model (1) can be expressed as a general linear model (Wawrzynek, 2009):

$$Y^T = (Y_1 Y_2 \dots Y_m) \quad (2)$$

$$\varepsilon^T = (\varepsilon_1 \varepsilon_2 \dots \varepsilon_n) \quad (3)$$

$$\beta^T = (\beta_1 \beta_2 \dots \beta_k) \quad (4)$$

$$f^T(x) = (f_1(x) f_2(x) \dots f_k(x)) \quad (5)$$

$$F = \begin{bmatrix} f_1(x_1) & \dots & f_k(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_n) & \dots & f_k(x_n) \end{bmatrix} \quad (6)$$

where  $f_i(x_j) \equiv x_{ij}$ , for  $i = 1, 2, \dots, k, j = 1, 2, \dots, n$ . Then the function defined as  $y = F\beta$  is referred to as a response surface function. In practical applications, it is typical to consider response surface functions that do not take account for interactions between factors, of the form (Wawrzynek, 2009; Montgomery, 2020):

$$y(x_1, x_2, \dots, x_m) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_m x_m \quad (7)$$

or response surface functions that account for factor interactions, given by the formula:

$$y(x_1, x_2, \dots, x_m) = \beta_0 + \beta_1 x_1 + \dots + \beta_m x_m + \beta_{12} x_1 x_2 + \dots + \beta_{m-1m} x_{m-1} x_m. \quad (8)$$

The analysis of experimental results is typically conducted using classical statistical techniques, such as analysis of variance and regression analysis. These methods require the use of parametric tests (Wawrzynek, 1993, 2009; Aczel, 2000; Kończak, 2007; Montgomery, 2009; Myers, Montgomery & Anderson-Cook, 2016). Additionally, the literature considers the use of permutation tests in the significance analysis of response surface function parameters (Złotoś, 2020).

Among the most commonly used experimental designs in practice are full factorial designs of experiments  $2^k$ , which require  $2^k$  experimental trials to be carried out. An important role in practical applications is played by fractional factorial designs of experiments  $2^{m-k}$ , that are constructed based on full factorial designs of experiments and lead to a reduction in the number of experiments by at least half (Dean, Voss & Draguljić, 2017; Rigdon *et al.*, 2022; Antony, 2023).

In the usual case, factorial designs of experiments refer to analysing the effect of a fixed number of factors on one response variable. In the case that the production process under study is characterised by two response variables, the analysis of the experimental results will address each variable individually. The recommendations made on the basis of the results obtained may then be inconclusive. In the literature, methods for analysing experimental results with two or more response variables are considered, which are based on graphical methods, take into account response surface functions with relevant parameters or desirability function (Derringer & Suich, 1980; Ryan, 2007; Boateng, 2023). However, the solution proposals considered in the literature do not take into account the nature of the response variables, including their ranges of variation.

### 3. Construction of Design of Experiment for Two Response Variables

An experiment involving  $k$  factors that requires  $n$  experimental trials to be performed will be considered. Two response variables,  $Y_1$  and  $Y_2$ , are considered in the study, for which the desired ranges of variation have been established as  $Y_1^{(z)} = [y_{1d}, y_{1g}]$  and  $Y_2^{(z)} = [y_{2d}, y_{2g}]$ , respectively. To determine the relationship between the factors considered and the response variables considered together, a synthetic variable must be constructed that takes into account the component variables  $Y_1$  and  $Y_2$ .

In the initial stage of the proposed method, for each realisation of the response variable in the individual experimental trials, a function of the form:

$$k(y_{ij}) = \begin{cases} 0, & y_{ij} \in Y_j^{(z)} \\ d(y_{ij}, Y_j^{(z)}), & y_{ij} \notin Y_j^{(z)} \end{cases} \quad (9)$$

is determined, where  $i = 1, 2, \dots, n, j = 1, 2$  and  $d(y_{ij}, Y_j^{(z)}) = \inf_{y \in Y_j^{(z)}} d(y_{ij}, y)$  is a distance of a point  $y_{ij}$  from the set  $Y_j^{(z)}$ . The function (9) characterises the position of the actual values of the component response variables relative to the desired ranges of variation. The values of the function of the form (9) are then normalised according to the formula:

$$z_{ij} = \frac{k(y_{ij})}{\max_i k(y_{ij})}. \quad (10)$$

In the second step of the proposed method, the values of the synthetic response variable are determined according to the following formula:

$$\tilde{y}_i = \delta z_{i1} + (1 - \delta) z_{i2}, \quad (11)$$

where  $\delta$  is a fixed quantity within the interval  $(0, 1)$ . The values of  $\delta$  and  $1 - \delta$  correspond to the weights of the individual component response variables  $Y_1$  and  $Y_2$ ,

and these can be set deliberately, depending on the specifics of the variables analysed. The defined synthetic variable forms the basis for further analysis of the experimental results.

The response surface function, which characterises the dependence of the synthetic response variable  $\tilde{Y}$  on the  $k$  factors considered in the experiment, is expressed by the following equation:

$$\tilde{y} = \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \dots + \gamma_k x_k + \tilde{\epsilon}. \quad (12)$$

The analysis of the experimental results, which involves the evaluation of the significance of the parameters of the response surface function of the form (12), can be carried out using classical methods (Montgomery, 2020) or permutation tests (Złotoś, 2020). It is important to acknowledge that permutation tests can be utilised in circumstances where the assumptions of classical statistical methods are not met or the sample size is limited (Kończak, 2016). The results obtained will then allow for the identification of those factors that have a significant impact on the values of the synthetic response variable.

The individual steps of the presented construction of a design of experiment for two response variables, due to computational complexity, have the potential to result in difficulties in implementing the method in practice. The utilisation of specialised statistical software is then recommended. Of particular note is the R programme (open source software), which facilitates not only the analysis of the results of classical designs of experiment (cf. Lawson, 2015), but also the implementation of permutation tests (cf. Kończak, 2016).

#### **4. The Application of the Proposed Method**

Wood pellets are classified as solid biofuels, which are currently the renewable energy source with the largest share of energy production in Poland (GUS, 2023). Wood pellet production technology is a complex process. Proper analysis of the wood pellet production process allows for the production of high-quality pellets in a way that provides economic benefits. The study will consider the wood pellet production process implemented in a certain company in the wood industry.

The objective of this study is to identify the factors that significantly influence the results of the production process of wood pellets produced by a specific company. Additionally, the experimenter's task was to determine the nature of the dependence of pellet quality on the aforementioned factors.

A comprehensive analysis of the various stages of the wood pellet production process was conducted, with a particular focus on identifying the factors that could potentially impact the quality of the final product. Only those factors for which there were no inherent measurement difficulties were included in the study. Four controlled factors were considered in the experiment:

- $X_1$  – moisture content of the raw material (%),
- $X_2$  – moisture content of the raw material after drying (%),
- $X_3$  – drying temperature ( $^{\circ}\text{C}$ ),
- $X_4$  – feeder speed (rpm).

In order to ascertain the range of variation and levels of the controlled factors, measurements were taken of the values of each factor during the production process. The resulting data is presented in graphical form in Figure 1.

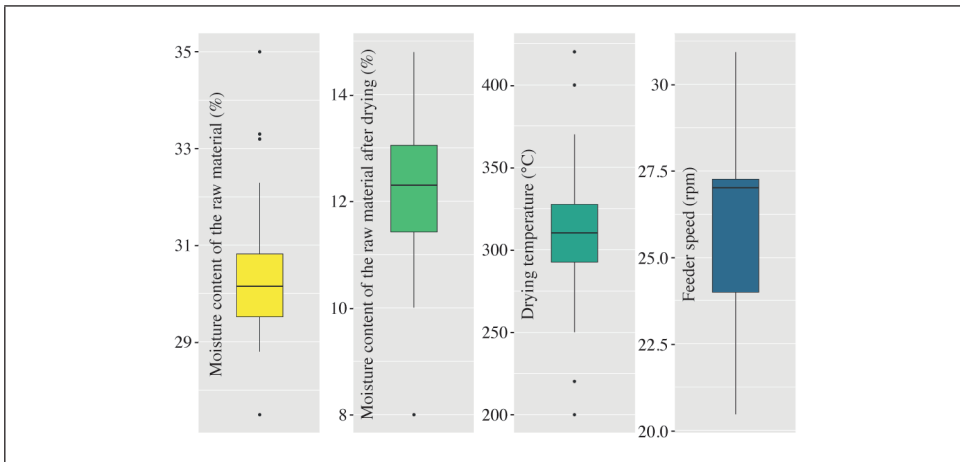


Fig. 1. Measurement Results of Individual Factors

Source: own elaboration.

The specific nature of the production process under study made it impossible to obtain precise values for the individual factor levels. An alternative method of determining factor levels was proposed, which refers to the proposal for imprecisely defined factors presented by Szerszunowicz (2011). Based on the measurements taken, a median value  $Me_i$  was determined for each factor ( $i \in \{1, 2, 3, 4\}$ ). It was agreed that each of the factors considered would be taken into account at two levels:

- upper level (denoted by “1”) is defined as a value of the factor  $X_i$  is greater than  $Me_i$ ,
- lower level (denoted by “-1”), when the value of the factor  $X_i$  is less than or equal to  $Me_i$ .

The subsequent step was to identify the response variable that most accurately reflects the pellet-making process. In the study under consideration, two response variables were identified:

- $Y_1$  – bulk density of wood pellets,
- $Y_2$  – production line efficiency.

The value for bulk density of pellets represents the weight of pellets (in kilograms) in a  $1 \text{ m}^3$  container. The efficiency of the production line is described by the weight of pellets produced (in kilograms) per hour. The response variables permit an assessment of the quality of the pellets produced and the efficiency of the pellet production process. The bulk density of wood pellets typically ranges from  $500 \text{ kg/m}^3$  to  $800 \text{ kg/m}^3$ , while the efficiency of the production line under consideration takes values from  $300 \text{ kg/h}$  to  $450 \text{ kg/h}$  (Kocsis & Csanády, 2019). Based on expert knowledge and accepted standards, ranges of values for the result variables were indicated, which would indicate the proper course of the wood pellet production process. It was assumed that pellets of optimal quality have a bulk density of  $680 \text{ kg/m}^3$  to  $720 \text{ kg/m}^3$ , while the capacity of the production line should be  $380 \text{ kg/h}$  to  $400 \text{ kg/h}$ .

The identification of the factors and the response variable enabled the selection of an appropriate experimental design. A full factorial experiment  $2^4$ , which would require the completion of 16 individual experimental trials, was initially considered. The specific characteristics of the pellet production process precluded the possibility of conducting such a large number of experimental trials in a short period of time and under identical conditions. Consequently, it was determined that a fractional factorial design of experiments  $2^{4-1}$  would be employed, with the understanding that individual experiments would be conducted randomly. The experimental design, including the sequence of experiments, is presented in Table 1. The established experimental design allows for a reduction in the number of experimental trials by half. The execution of the experiments is then carried out under as uniform conditions as possible, which allows for a relative reduction in the influence of interfering factors. Furthermore, reducing the number of experiments leads to a reduction in the execution time of the experiment. This reduces the costs associated with the operation of the production line and possible downtime in the production process.

Table 1. Scheme for the Realisation of Fractional Factorial Design Experiments  $2^{4-1}$

Number of Experimental Trial (Sequence)	$X_1$	$X_2$	$X_3$	$X_4$
1 (8)	1	1	1	1
2 (3)	1	1	-1	-1
3 (5)	1	-1	1	-1
4 (1)	1	-1	-1	1
5 (6)	-1	1	1	-1
6 (2)	-1	1	-1	1
7 (4)	-1	-1	1	1
8 (7)	-1	-1	-1	-1

Source: own elaboration.

The determination of the levels of factors and response variables and the choice of the experimental design allowed the design of experiments to be carried out in a specific order. The results of the conductive of experiments are presented in Table 2.

Table 2. The Results of Experimental Trials

Number of Experimental Trial (Sequence)	$X_1$	$X_2$	$X_3$	$X_4$	$Y_1$	$Y_2$
1 (8)	1	1	1	1	714	468
2 (3)	1	1	-1	-1	730	362
3 (5)	1	-1	1	-1	696	372
4 (1)	1	-1	-1	1	850	354
5 (6)	-1	1	1	-1	694	318
6 (2)	-1	1	-1	1	512	348
7 (4)	-1	-1	1	1	610	330
8 (7)	-1	-1	-1	-1	802	336

Source: own elaboration.

The aim of the present experiment was to determine the dependence of the response variables on individual factors. Two response variables were included in the study, and therefore two response surface functions were considered of the form:

$$y^{(1)} = \beta_0^{(1)} + \beta_1^{(1)}x_1 + \beta_2^{(1)}x_2 + \beta_3^{(1)}x_3 + \beta_4^{(1)}x_4 + \varepsilon^{(1)}, \quad (13)$$

$$y^{(2)} = \beta_0^{(2)} + \beta_1^{(2)}x_1 + \beta_2^{(2)}x_2 + \beta_3^{(2)}x_3 + \beta_4^{(2)}x_4 + \varepsilon^{(2)}. \quad (14)$$

In order to determine the dependence  $Y_1$  of the response variable on the factors considered, the coefficients of the response surface function of the form (13) were estimated and their significance was verified. Due to the small number of observations, the evaluation of the significance of the parameters of the response surface function was carried out using appropriate permutation tests (Złotoś, 2020). The results of these analyses are presented in Table 3.

For each of the estimated parameters, the value of  $A\hat{S}L$  is greater than the established significance level, so there is not enough evidence to reject the null-hypothesis that the factors do not influence the values of the response variable  $Y_1$ .

Similar considerations were made for the response variable  $Y_2$ . The results of the permutation tests performed to verify the significance of the estimated parameters of the response surface function of the form (14) are presented in Table 4.

Table 3. Results of Testing Significance of Response Surface Function (13)

Parameter $\beta_i^{(1)}$	Parameter Estimator Value $\beta_i^{(1)}$	A $\hat{S}L$ Value
$\beta_0^{(1)}$	701.00	0.5981
$\beta_1^{(1)}$	46.50	0.3179
$\beta_2^{(1)}$	-38.50	0.3833
$\beta_3^{(1)}$	-22.50	0.5917
$\beta_4^{(1)}$	-29.50	0.5202

Source: own elaboration.

Table 4. Results of Testing Significance of Response Surface Function (14)

Parameter $\beta_i^{(2)}$	Parameter Estimator Value $\beta_i^{(2)}$	A $\hat{S}L$ Value
$\beta_0^{(2)}$	356.25	0.5089
$\beta_1^{(2)}$	23.25	0.1976
$\beta_2^{(2)}$	8.25	0.8643
$\beta_3^{(2)}$	15.75	0.4988
$\beta_4^{(2)}$	18.75	0.3631

Source: own elaboration.

As in the case of the response variable  $Y_1$ , the results obtained do not allow the identification of factors that have a significant impact on the values of the response variable  $Y_2$ . Therefore, the analysis of the experimental results using permutation tests did not allow any recommendations to be made for the setting of the parameters of the wood pellet production process under consideration.

It should be noted that the analysis of the experimental results carried out referred to each of the response variables considered individually. In practice, this may lead to inconclusive results (e.g., a factor has a significant effect on only one of the variables considered). In addition, the study did not take into account the desired ranges of values for the response variables, which were determined at the design stage of the experimental design. This means that methods should be sought to construct a design that allows the analysis of experimental results that take into account more than one response variable at a time. In particular, consideration should be given to response variables whose values are expected to fall within the desired ranges of variability.

The study considered two response variables: bulk density ( $Y_1$ ) and production line efficiency ( $Y_2$ ). For these variables, the desired ranges of variation were

determined:  $Y_1^{(z)} = [680, 720]$  and  $Y_2^{(z)} = [380, 400]$ , respectively. According to the proposed method of analysis of the experimental results, for each of the obtained values of the result variables, the values of the function (9) were determined, which for the considered process takes the forms:

$$k(y_{i1}) = \begin{cases} 0, & y_{i1} \in Y_1^{(z)} \\ d(y_{i1}, Y_1^{(z)}), & y_{i1} \notin Y_1^{(z)} \end{cases} \quad (15)$$

and

$$k(y_{i2}) = \begin{cases} 0, & y_{i2} \in Y_2^{(z)} \\ d(y_{i2}, Y_2^{(z)}), & y_{i2} \notin Y_2^{(z)} \end{cases} \quad (16)$$

where  $i = 1, 2, \dots, 8$ . The values obtained were then normalised according to formula (10). In addition, the values of the synthetic response variable were determined according to formula (11). Due to the similar importance of the considered response variables, the value of  $\delta = 0.5$  was taken. The normalised values of functions (15), (16) and the synthetic response variable for each experiment are shown in Table 5.

Table 5. The Normalised Values of Functions (15) and (16) and Value of Synthetic Response Variable

Number of Experimental Trial (Sequence)	$X_1$	$X_2$	$X_3$	$X_4$	$z_{i1}$	$z_{i2}$	$\tilde{y}_i$
1 (8)	1	1	1	1	0.0000	1.0000	0.5000
2 (3)	1	1	-1	-1	0.0595	0.8235	0.4415
3 (5)	1	-1	1	-1	0.0000	0.1176	0.0588
4 (1)	1	-1	-1	1	0.7738	0.3824	0.5781
5 (6)	-1	1	1	-1	0.0000	0.9118	0.4559
6 (2)	-1	1	-1	1	1.0000	0.4706	0.7353
7 (4)	-1	-1	1	1	0.4167	0.7353	0.5760
8 (7)	-1	-1	-1	-1	0.4881	0.6471	0.5676

Source: own elaboration.

The theoretical values of the synthetic response variable depend on the values of the component variables and are shown in Figure 2. It should be noted that the results of the production process will be of high quality if the values of the synthetic response variable are as small as possible.

A response surface function will be considered that determines the dependence of the synthetic response variable  $\tilde{Y}$  on the factors of the form:

$$\tilde{y} = \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 + \tilde{\epsilon}. \quad (17)$$

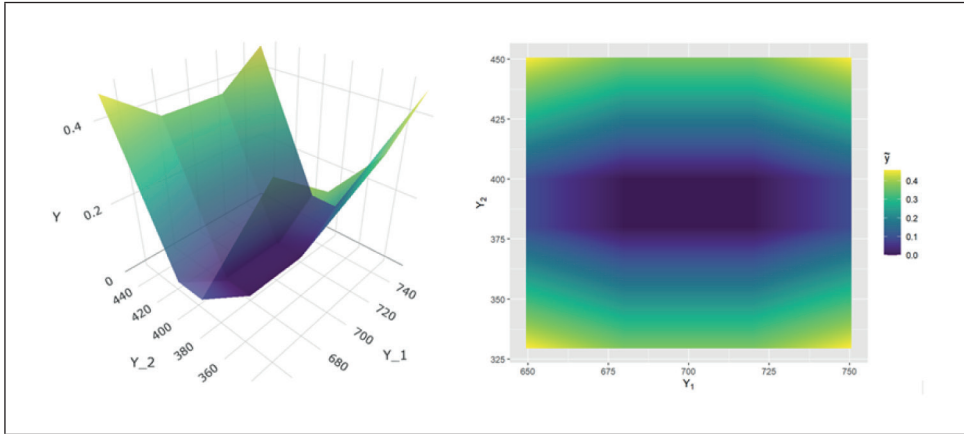


Fig. 2. Theoretical Values of the Synthetic Response Variable  
 Source: own elaboration.

The analysis of the established response surface function was conducted using permutation tests. The results of the response surface function parameter estimates (17) and an assessment of their significance are presented in Table 6.

Table 6. Results of the Response Surface Function Analysis (17)

Parameter $\gamma_i$	Parameter Estimator Value $\gamma_i$	$A\hat{S}L$ Value
$\gamma_0$	0.4891	0.0154
$\gamma_1$	-0.0945	0.0188
$\gamma_2$	0.0440	0.4126
$\gamma_3$	-0.0915	0.0261
$\gamma_4$	0.1082	0.0088

Source: own elaboration.

The resulting  $A\hat{S}L$  value estimates are below the established significance level  $\alpha = 0.05$  for the parameters  $\gamma_0, \gamma_1, \gamma_3, \gamma_4$ . This indicates that the values of the synthetic response variable  $\hat{Y}$  are significantly influenced by factors  $X_1, X_3$  and  $X_4$ . In order to identify the settings of the factors that will result in the lowest values for the response variable, it is necessary to consider the response surface function, which is given by the following equation:

$$\hat{y} = 0.4891 - 0.0945x_1 - 0.0915x_3 + 0.1082x_4. \tag{18}$$

Consequently, for the factor  $X_4$  at the lower level, the smallest value of the synthetic response variable will be obtained when the factors  $X_1$  and  $X_3$  are set at the

upper levels. The dependence of the value of the response surface function (18) on factor settings  $X_1$  and  $X_3$ , for  $X_4 = -1$  are shown in Figure 3.

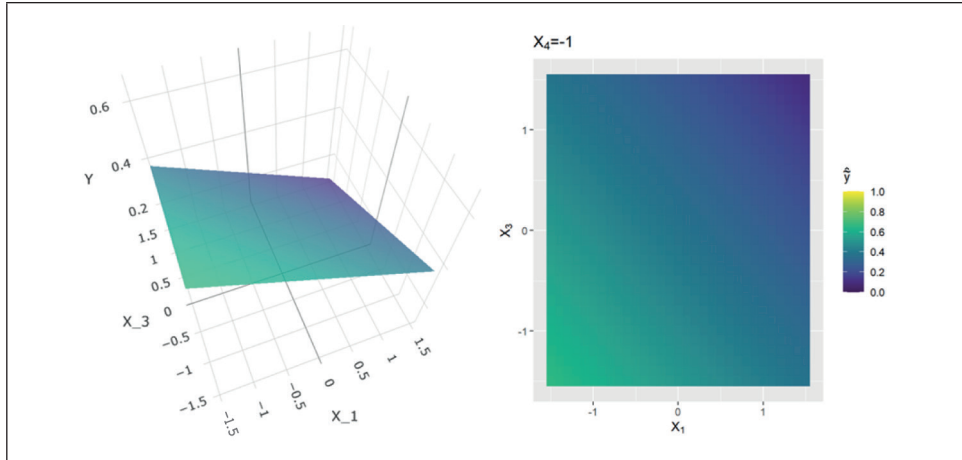


Fig. 3. Response Surface Function Values (18) for  $X_4 = -1$

Source: own elaboration.

When a factor  $X_4$  is set at the upper level, the smallest value of the synthetic response variable will be achieved for factors  $X_1$  and  $X_3$  set at the upper levels. The values of the response surface function (18) as a function of factor levels  $X_1$  and  $X_3$  are illustrated in Figure 4. It should be noted that the best of the two factor settings indicated would be to set the factor  $X_4$  at the lower level and the factors  $X_1$  and  $X_3$  at the upper levels.

The results obtained for the wood pellet production process under consideration permit the assertion that the bulk density of pellets and the efficiency of the production line are significantly dependent on the moisture content of the raw material (factor  $X_1$ ), drying temperature (factor  $X_3$ ) and on the feeder speed (factor  $X_4$ ). In addition, it is possible to obtain values for bulk density and production line efficiency that fall within the respective desired value ranges.

In conclusion, a recommendation was made for the production process under consideration, which states that obtaining good quality pellets involves:

- the use for production of raw material with a moisture content above 30.15%,
- setting the drying temperature above 310°C,
- setting the feeder speed to a level less than or equal to 27 rpm.

The above factor settings provide the best opportunity to obtain values for bulk density and production line efficiency at levels corresponding to the desired ranges of variation.

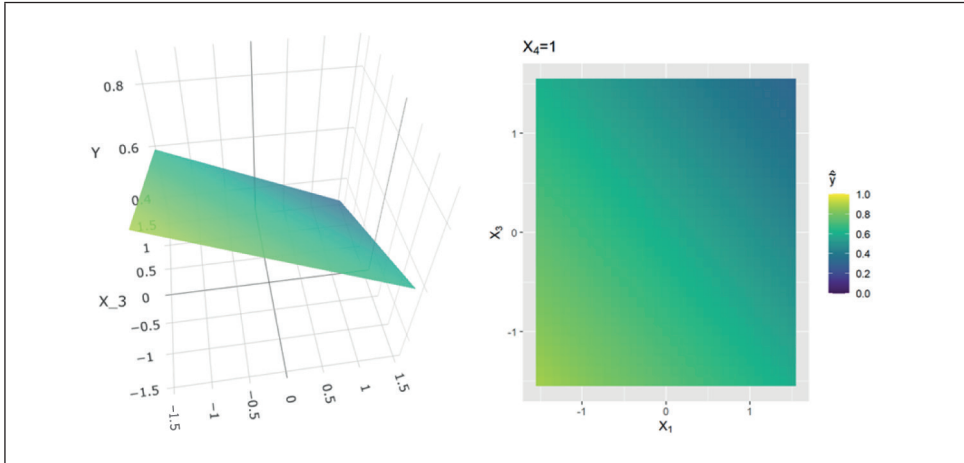


Fig. 4. Response Surface Function Values (18) for Factor  $X_4 = 1$

Source: own elaboration.

The presented method has been effectively used to enhance the efficiency of the production line and the quality of the pellets produced. It is noteworthy that the proposed method has the potential for broader implementation in the context of industrial practice. With particular reference to enterprises specialising in the implementation of manufacturing or chemical processes, the results of which are characterised by two response variables. Furthermore, the proposed design of experiment incorporates the desired ranges of values for the outcome variables, thereby enabling the consideration of the specificities resulting from the recommendations for the process under study. The employment of permutation tests in the aforementioned approach renders this method suitable for the analysis of production processes, the course of which prevents the conduct of numerous experimental trials or their replication (which is usually required by classical or factorial designs of experiments).

## 5. Conclusions

This article presents the author’s proposal for the construction of an experimental plan that allows the analysis of the influence of a fixed number of factors simultaneously on two outcome variables. Furthermore, the presented design of the experiment plan takes into account the desired ranges of variation for the values of the individual outcome variables.

The proposed method represents an alternative to the few approaches presented in the existing literature. Typically, these methods rely on individual analysis of each of the outcome variables considered, refer to graphical methods or use a suitably

specified desirability function in their construction (cf. Boateng, 2023). However, the indicated methods do not take into account established ranges of variability for the values of the considered outcome variables, which may lead to inconclusive conclusions for the considered production process. Furthermore, the analysis of experimental results is carried out using parametric methods, the use of which is not always justified.

The proposed method involves determining the value of an appropriately constructed synthetic variable. This permitted the analysis of a single response surface function, which took into account the two outcome variables and their ranges of variation. Moreover, the analysis of the experimental results was conducted using permutation tests. The implementation of the proposed method in the wood pellet production process enabled the formulation of clear technological recommendations to be made for the investigated process.

### Conflict of Interest

The author declares no conflict of interest.

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