Zeszyty Naukowe Uniwersytet Ekonomiczny w Krakowie

10 (970)

ISSN 1898-6447
Zesz. Nauk. UEK, 2017; 10 (970): 55–66
DOI: 10.15678/ZNUEK.2017.0970.1004

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# Producers' Optima in Schumpeterian Evolution*

**Abstract**

The paper presents a production system in the Debreu model of general equilibrium. According to Schumpeter, economic development is possible only on the strength of innovations being introduced. This process provides a sequence of optimal production plans, corresponding to each stage of the innovative evolution. The paper characterises the sequence of optimal plans and provides the conditions for its convergence. Moreover, the limiting production plan is shown to be the producer's optimum in the final state.

**Keywords**: Debreu model, production system, Schumpeterian evolution, innovation, Kuratowski convergence.
**JEL Classification:** C61, C62, D50.

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* This publication presents the results of a research project financed from a subsidy for the maintenance of research potential, granted to the Faculty of Finance and Law at Cracow University of Economics.

# 1. Introduction

*To the memory of Professor Andrzej Malawski*

The paper examines the production system of the Debreu economy and focus on its Schumpeterian evolution. The goal of a producer is to maximise profit over the set of production. The success of all market agents is possible if and only if there exists a price system common to everybody. In the second part of the paper, the formal description of the production system and the framework of the Debreu model are briefly reviewed.

J. A. Schumpeter (1912) determined two forms of economic life: circular flow and economic development. The first form corresponds to a state when all the processes and agents follow the known economic rules, even if some new good appears in the market. The second form, economic development, can be obtained via creative destruction. We review the basic facts from this theory in the second part of the paper. In the same part we briefly recall some results from the papers of B. Ciałowicz and A. Malawski (2011) and A. Lipieta and A. Malawski (2016), who attempted to describe formally the Schumpeterian mechanisms. This is done to justify the approach in the third part, where we use the fact that in the production system of the Debreu model the Schumpeterian evolution is observed via changing sets of production. However, the above authors' works define the cumulative and innovative extensions of producer's system, requiring in the definitions that the producers must increase their profits. One question remains open here: whether or not a producer who in each stage of the transitions of the economy maximises his profit will ultimately maximise their profit. The main goal of the paper is to answer this question.

The main results of this paper (theorems 1 and 2) include: in Schumpeterian evolution, producers are not only better off (as definitions established by A. Malawski and B. Ciałowicz emphasised), but also the final best production plan may be achieved by realising, step-by-step, the best production plans in the particular stages of evolution.

The results are based on previously obtained theorems for the linear programming problem we described in terms of the Kuratowski convergence (see Denkowska, Denkowski & Kornafel 2017). The next two parts briefly present the definitions and summarise the main properties of this kind of convergence of sets and comment on the requisite mathematical theorems.

The work done for this paper has only a theoretical character and is intended to complete the mathematical description of economic theory of the Schumpeterian evolution in the Debreu model. However, practitioners may also profit from our results: Mathematically, the problem of maximising the producers' profits over sets, which are polytopes, is a linear programming problem. A well-known numerical method of solving it is called the simplex method, which has a rich bibliography (see e.g. Bertsimas, Tsitsiklis & Tsitsiklis 1997, Bartels & Golub 1969, Karmarkar 1984). The practical conclusion of our result is the following: a convergent sequence of optimal production plans (found in each stage of evolution by the simplex method) has a limit that is the optimum for limiting the problem. Secondly, despite the possible numerical errors in computations, the calculated result is *close* to actual optimum.

## 2. Production System in the Debreu Model

In his monumental 1959 work, G. Debreu described the general equilibrium model using a strictly mathematical apparatus. The linear space $\mathbb{R}^l$ (with fixed $l \in \mathbb{N}$) is interpreted as the $l$-dimensional space of commodities and prices. The model formally consists of production and consumption systems, determined by the behaviour of two groups of agents: producers $j \in B$ and consumers $i \in C$ (with a finite number of members in each group). The consumer is characterised by his preferences and budget, his goal being to maximise utility from consumption over the budget set. The $j$th producer is described by his production set $Y_j \subset \mathbb{R}^l$ (determining the production abilities and available technology) and aims to maximise his profit, i.e.:

$$\pi_j^* := \max \pi_j = \max_{y_j \in Y_j} p \cdot y_j .$$

Mathematically, it is a problem of maximising the linear functional $p \cdot y_j$ over the given set $Y_j$. Moreover, when the set $Y_j$ is a cone (as we see in a moment in the assumptions), the maximisation problem is merely simple linear programming problem.

G. Debreu joins those two sectors and provides the conditions under which it is possible to achieve equilibrium. The assumptions for the production system, which is the heart of the matter here, are reviewed below.

For any producer $j$, production set $Y_j$ satisfies the following conditions (see Debreu 1959):

a) $Y_j \subset \mathbb{R}^l$ is closed (meaning that if for any the production plan $y_j^n \in Y_j$ is possible for $j$th producer and $\lim_{n\to\infty} y_j^n = y_j^0$, then $y_j^0 \in Y_j$, i.e. the limiting plan can also be realised),

b) $0 \in Y_j$ (the possibility of not producing),

c) $Y_j \cap \mathbb{R}_+^l \subset \{0\}$ (i.e. free production is impossible without inputs),

d) $Y_j \cap (-Y_j) \subset \{0\}$ (irreversibility),

e) $Y_j + Y_j \subset Y_j$ (any two production plans together are also possible for production),

f) $Y_j$ is convex (any combination of two production plans is also possible for production),

g) $Y_j$ is a cone with its vertex at (under the assumption of constant returns to scale),

h) $\mathbb{R}^l_- \subset Y$, where $Y = Y_1 + \cdots + Y_n$ (it is possible for all producers together to dispose of all commodities).

As we said, for a given price system $p \in \mathbb{R}^l$, a producer aims to maximise profit. It is known that maximisation may not be possible for any price system, so the following is defined:

$$T_j := \left\{ p \in \mathbb{R}^l \text{: there exists } \max_{y_j \in Y_j} (p \cdot y_j) \right\}.$$

For a production set that is a cone, the set $T_j$ is its normal cone. The correspondence attaining the maximisers to the given price $p \in T_j$ is:

$$\eta_j \colon \mathbb{R}^l \ni p \mapsto \left\{ y_j \colon p \cdot y_j = \max_{y_k \in Y_k} (p \cdot y_k) \right\} \subset Y_j .$$

In other words, for $p \in T_j$ $n_j(p) = \arg\max \pi_j(p)$. In general, if $Y_j$ is a polytope, the set $\eta_j(p)$ can be identified with the set of vertices of the production set.

In such a setting, if only $\bigcap_{j \in B} T_j \neq \varnothing$, it is possible to prove the existence of the price system, which allows all the producers in the economy to maximise their profits and – in the further perspective of the model and with additional requirements toward the consumption system – the existence of equilibrium in the whole economy. The assumptions listed above remain in force in the next parts of the paper.

## 3. Schumpeterian Evolution

In his 1912 work, J. Schumpeter distinguished two forms of economic life: circular flow and economic development. Circular flow is the state of the economy, in which all the processes go along known trajectories, as determined by economic laws. It could be understood as stagnation. However, an economy undergoes constant evolution, which to J. Schumpeter meant evolution is driven by creative destruction, or the natural process of introducing innovations and eliminating existing goods, production technologies, markets, etc. In 1950, J. Schumpeter wrote: “The fundamental impulse that acts and keeps the capitalistic engine in motion comes from the new consumer goods, the new method of production, the new forms of industrial organization that capitalist enterprise creates.

(…) The opening up of new markets, foreign or domestic, and the organizational development (…) illustrate the same process of industrial mutation – if I may use that biological term – that revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one" (Schumpeter 1950, p. 83).

J. Schumpeter never explained how creative destruction changes circular flow. With his research group at Cracow University of Economics and colleagues from other countries A. Malawski, in a number of publications, has attempted to describe formally how it would happen (see *Innovative Economy…* 2013) and references therein). In particular, B. Ciałowicz and A. Malawski (2011) introduced the definitions of cumulative and innovative extensions of a production system. In the definitions cited below, the "tilde" symbol denotes the quantities, functions and correspondence after the change introduced.

*Definition 1* (Ciałowicz & Malawski 2011). A production system $\tilde{P}=(\tilde{B},\mathbb{R}^{\tilde{l}},\tilde{y},\tilde{p},\tilde{\eta},\tilde{\pi})$ is called a cumulative extension of a production system $P=(B,\mathbb{R}^{l},y,p,\eta,\pi)$, briefly $P\subset_c\tilde{P}$, if:

1) $l\leq\tilde{l}$;

2) $p\leq proj_{\mathbb{R}^l}(\tilde{p})$, where *proj* denotes orthogonal projection and the inequality between vectors is understood as:

$$p\leq q\Leftrightarrow\forall i:p_i\leq q_i;$$

3) $B\subset\tilde{B}$ and for every $b\in B$:
   a) $Y_j\subset proj_{\mathbb{R}^l}(\tilde{Y}_j)$,
   b) $\eta_j\subset proj_{\mathbb{R}^l}(\tilde{\eta}_j(\tilde{p}))$,
   c) $\pi_j(p)\leq\tilde{\pi}_j(\tilde{p})$.

This definition describes the situation when creative destruction is obtained via the creation of a new good (condition 1, which describes possible extension of the dimension of the commodity and price spaces) or via the introduction of a new technology (condition 3a) and the presence of a new market agent (if $B\neq\tilde{B}$), who actually might be the source of the two listed previously. In particular, if $l=\tilde{l}$ and $B=\tilde{B}$, the projections are identity mappings, so a change may be obtained only by extending the production set $\tilde{Y}_j$, but all the previously used technologies are still in use. This extension may be done, for instance, by acquiring additional machines. The whole economic environment therefore remains unchanged and the cumulative extension is intended to model the circular flow in economics. B. Ciałowicz and A. Malawski (2011) also defined a strong cumulative extension of the production system, considering the extensions with respect to different aspects (like number of commodities, price system, etc.) and emphasising the character of circular flow in them.

*Definition 2* (Ciałowicz & Malawski 2011). A production system $\tilde{P}=(\tilde{B},\mathbb{R}^{\tilde{l}},\tilde{y},\tilde{p},\tilde{\eta},\tilde{\pi})$ is called an innovative extension of a production system $P=(B,\mathbb{R}^{l},y,p,\eta,\pi)$, briefly $P\subset_i\tilde{P}$, if:

1) $l\leq\tilde{l}$;
2) $p=proj_{\mathbb{R}^l}(\tilde{p})$;
3) $\exists\tilde{j}\in\tilde{B}\ \ \forall j\in B$:
   a) $proj_{\mathbb{R}^l}(\tilde{Y}_{\tilde{j}})\not\subset Y_j$,
   b) $proj_{\mathbb{R}^l}(\tilde{\eta}_{\tilde{j}})(\tilde{p})\not\subset\eta_j(p)$,
   c) $\pi_j(p)\leq\tilde{\pi}_{\tilde{j}}(\tilde{p})$.

*Remark*. The producers who satisfy condition 3 are called innovators.

The definition of innovative extension emphasises the presence of an innovator $\tilde{j}$, who may introduce a new good or open a new market (if $l<\tilde{l}$ in condition 1), or may introduce a new technology (method) of production (condition 3a). In contrast to the cumulative extension, the innovative extension exacts the introduction of a new good or new technology and it is possible to rule out an unnecessary or no longer productive technology from the new set of production $\tilde{Y}_{\tilde{j}}$.

A. Lipieta and A. Malawski (2016) proposed to design economic mechanisms to describe an economic system's evolution from an arbitrary one to its cumulative extension (price-preserving mechanism) and evolution to the innovative extension (qualitative mechanism). They study the relationship between these extensions and use the language of mechanism design developed by L. Hurwicz and S. Reiter (2006). One of the elements of their description is the transition mapping $T:[0,1]\rightarrow\wp_j$, where $T(0)=P_j$ and $T(1)=\tilde{P}_j$. The set $\wp_j$ denotes all the possible production systems. The authors showed (Lipieta & Malawski 2016) that it is possible to design transitions that will lead to cumulative and innovative extensions, respectively.

A question that does not appear in the analysis is whether or not a producer who in each stage of those transitions maximises his profit will ultimately maximise profit. From a mathematical point of view, the answer is not obvious. Definitions 1 and 2 assure only that the final profit will not be smaller than it was at the beginning. In the next part of the paper, a theorem showing that this convergence is assured is formulated using the language of Kuratowski convergence, which makes it possible to describe interesting phenomenon relatively easily. To clarify the issue, the necessary mathematical definitions and theorems are written down in the 5th and 6th sections.

## 4. The Dynamics of the Producers' Optima in Schumpeterian Evolution

Let us consider the mapping (transition) $T:[0,1]\to \wp_j$, which describes the evolution of the production system $P_j=T(0)$ to another state $\tilde{P}_j=T(1)$. In particular, the final state may be a cumulative or innovative extension of the production system $P_j$.

For any time moment $t\in[0,1]$ we deal with the production system $P_j^t=(B,\mathbb{R}^{l^t},y^t,p^t,\eta^t,\pi^t)$, in which the $j$th producer maximises his profit $\pi_j^t$ over the production set $Y_j^t$. The price vectors $(p^t)_t$ by assumption create a convergent sequence and $\lim_{t\to 1}p^t=:p^1$. The prices $p^t$ and $p^1$ are assumed to be such vectors that and $\eta_j^t(p^t)\neq\varnothing$ and $\eta_j^1(p^1)\neq\varnothing$. Then, as shown by arguments from the Debreu model, there exists $y_j^{t*}\in\eta_j^t(p^t)$.

In the theorems cited in part 6, all the sets are subsets of the same vector space $\mathbb{R}^l$. Here, due to the important role of dimension (determining the number of goods in the market), constant $l$ is quite an unpleasant inconvenience and would limit us only to the case when the dimension is not increased in the Schumpeterian evolution. We overcome this difficulty by the following argument. When $(l^t)$ is a non-constant sequence, we assume it to be bounded (which is economically reasonable as it is impossible to create or develop an infinite number of goods or technologies in a given time period). It is then enough to define $l:=\sup_{t\in[0,1]}l^t$ and identify the vectors and sets from $\mathbb{R}^{l^t}$ with their embeddings in $\mathbb{R}^l$. To be clear:

1) the vector $p^t\in\mathbb{R}^{l^t}$ will be identified with the vector

$$\tilde{p}:=p^t\times\underbrace{\{0\}\times\ldots\times\{0\}}_{l-l^t\text{ times}}\in\mathbb{R}^l,$$

2) the production set $Y_j^t\subset\mathbb{R}^{l^t}$ will be identified with the set

$$\tilde{Y}_j^t:=Y_j^t\times\underbrace{\{0\}\times\ldots\times\{0\}}_{l-l^t\text{ times}}\in\mathbb{R}^l,$$

3) the correspondence $\eta_j^t(p^t)\subset\mathbb{R}^{l^t}$ will be identified with the set

$$\tilde{\eta}_j^t(\tilde{p}):=\eta_j^t(p^t)\times\underbrace{\{0\}\times\ldots\times\{0\}}_{l-l^t\text{ times}}\in\mathbb{R}^l.$$

In such a setting, it is immediately clear from the definition of the canonical scalar product of vectors that the value of the profit function does not depend on whether we consider the original elements $p^t$ and $y_j^{t*}$ or the corresponding elements from the embeddings.

By assumption (g) in the Debreu model with the constant returns to scale (see part 2), production sets $Y_j^t$ are cones, and therefore special cases of polytopes with one vertex and two faces.

*Theorem 1.* Assume that $\lim_{t\to 1} Y_j^t = Y_j^1$ in the Kuratowski sense. Then:
1) $Y_j^1$ is a cone,
2) there exists a subsequence of the sequence of vertices of sets $Y_j^t$, i.e. the sequence of vectors $(v_j^t)$, which is convergent to the vertex $v_j^1$ of set $Y_j^1$.

Proof. As mentioned above, cones are special cases of polytopes with one vertex. There is therefore, obviously, $\#Y_j^t = 1$, so the second condition from the theorem 3 is satisfied. This theorem immediately provides the second part the statement 2 in theorem 1. Because $\#Y_j^t = 1$ and the set is a polytope, it is clear that it is also a cone, which completes the proof of the statement 1 in theorem 1. □

The first theorem shows that the Kuratowski limit of the sequence of production sets is a good candidate for the production system in the final state. The optimal plans are then indeed convergent to the optimal plan in the final system. This theorem characterises very well the convergence for both cumulative and innovative extensions of the production system.

*Theorem 2*. The sequence of sets $\left(\eta_j^t(p^t)\right)_t$ is convergent to $\eta_j^1(p^1)$ in the Kuratowski sense.

Proof. The theorem is the consequence of theorem 4. If necessary, the economic system is considered in the space $\mathbb{R}^l$ with $l := \sup_{t\in[0,1]} l^t$, as described above. □

In the second theorem, the convergence of maximisers is shown also in the case of price evolution. Recall that this was present in the considerations on circular flow and cumulative extensions of the production systems.

## 5. Mathematical Appendix: Kuratowski Convergence

In this part of the paper we are going to recall briefly the definition of the Kuratowski limit of a sequence of sets. Due to the fact that our economic model in is set in $n$-dimensional real space $\mathbb{R}^n$, the definition and basic facts are formulated in the simplified version of this vector space. Those results are known and the reader may find more details in (Dal Maso 1993).

The Kuratowski convergence is a generalisation of the convergence in the Hausdorff metric to the convergence of closed sets.

Hausdorff sought to extend his metric (defined for nonempty compact subsets of a metric space) to closed sets. In the beginning of the 20th century, P. Painlevé introduced the concept of upper and lower limits of a sequence of closed subsets in a metric space. The resulting convergence was later studied by several mathematicians (L. Zoretti, C. Zarankiewicz) but it was K. Kuratowski who first prepared a thorough exposition of this theory in his monumental book *Topologie* (Kuratowski 1961). It soon became apparent that this natural convergence of closed sets is a most useful tool for optimisation. In particular, the famous De Giorgi's Γ-convergence of extended-valued functionals on a topological space is precisely the Kuratowski convergence of their epigraphs. It is a powerful variational convergence in that both the minima and minimisers converge to the minimum (respectively, the minimiser) of the limiting functional.

Recall the basic definitions and facts. Let $E \subset \mathbb{R}^k \times \mathbb{R}^n$ be a nonempty set and $\pi\colon \mathbb{R}^k \times \mathbb{R}^n \ni (t,x) \mapsto t \in \mathbb{R}^k$ the natural projection. Fix an accumulation point $t_0 \in \pi(E) \setminus \{t_0\}$. We write $E_t = \{x \in \mathbb{R}^n\colon (t,x) \in E\}$ for the section of $E$ at $t$.

*Definition 3*. We define the lower and upper Kuratowski limits of the family (net) $(E_t)$ when $t \to t_0$ when respectively as the sets:

$$x \in \liminf_{t \to t_0} E_t \Leftrightarrow \text{for any neighbourhood } U \text{ of } x, \text{there is a neighbourhood } V \text{ of } t_0$$

$$\text{such that } U \cap E_t \neq \varnothing, \qquad \text{for all } t \in V \cap \pi(E) \setminus \{t_0\};$$

$$x \in \limsup_{t \to t_0} E_t \Leftrightarrow \text{for any neighbourhood } U \text{ of } x \text{ and any neighbourhood } V \text{ of } t_0$$

$$\text{there is a point } t \in V \cap \pi(E) \setminus \{t_0\} \quad \text{such that } U \cap E_t \neq \varnothing.$$

Clearly, $\liminf_{t \to t_0} E_t \subset \limsup_{t \to t_0} E_t$. If the converse inclusion also holds, we denote the resulting set by $\lim_{t \to t_0} E_t$ and call it the Kuratowski limit of $E_t$ when $t \to t_0$. Therefore, $(E_t)$ converges to some set $F$ as $t \to t_0$, iff

$$\limsup_{t \to t_0} E_t \subset F \subset \liminf_{t \to t_0} E_t.$$

We also then write that $E_t \xrightarrow{K} F$ (when $t \to t_0$).

It is easy to see that the upper and lower limits are closed sets that remain unchanged, if we replace the sets $E_t$ by their closures.

Observe that a sequence of sets $E_\nu \subset \mathbb{R}^n$ may be treated as the $t$-sections of the set $E := \bigcup_{\nu=1}^{+\infty} \left\{\frac{1}{\nu}\right\} \times E_\nu \subset \mathbb{R} \times \mathbb{R}^n$. Then the convergence of the sequence $(E_\nu)$ when $\nu \to +\infty$ is simply the convergence of the sections $(E_t)$ when $t \to 0$. Clearly, $\liminf_{\nu \to +\infty} E_\nu$ consists in this case of all the possible limits of convergent sequences chosen point by point: $x_\nu \in E_\nu$, while $\limsup_{\nu \to +\infty} E_\nu$ is the set of all the limits of convergent subsequences $x_{\nu_k} \in E_{\nu_k}$.

The most important feature of this convergence is that in the setting introduced above it is metrisable and compact (C. Zarankiewicz).

## 6. Mathematical Appendix: Convergence of Optima in a Linear Programming Problem

In linear programming, the vertices of a given linear polytope play an important role. As a matter of fact, if a linear functional attains its extremum on a linear polytope, then it attains it on the boundary and in particular in a vertex (provided, of course, that the set of extremal points $E^*$, i.e. vertices, is nonempty). Geometrically, this is captured by the position of the gradient of the functional with respect to the normal cones computed at the vertices.

One natural question is what happens when we allow some evolution in time (either of the constraints, i.e. of the polytope, or of the functional). Theorem 3 below gives a sufficient condition for the limit set of a Kuratowski-convergent sequence of linear polytopes to be a linear polytope. Moreover, we prove that the normal cones at the vertices converge to normal cones. In the non-compact case, the assumption of a uniform bound on the number of extremal points is not sufficient to obtain a polytope as a limit (see our paper Denkowska, Denkowski & Kornafel 2017).

*Theorem 3*. Assume that $\lim_{\nu\to+\infty} E_\nu = E$, where $E_\nu \subset \mathbb{R}^n$ are linear polytopes. Let $f_k(E_\nu)$ denote the number of $k$-dimensional faces of the polytope $E_\nu$ (here $k \in \{0, \ldots, \dim E_\nu\}$, where the dimension is the dimension of the affine envelope).

If there is a constant $M > 0$ such that either $f_{\dim E_\nu - 1}(E_\nu) \leq M$ for all $\nu$, or $E$ is compact and $\# E_\nu^* \leq M$, then:

1) $E$ is a linear polytope, too,

2) $f_{\dim E - 1}(E) \leq M$ or $\# E^* \leq M$, respectively,

3) for any $a \in E^*$ there is a sequence $a_\nu \in E_\nu^*$ converging to such that the normal cones $N_{a_\nu}(E_\nu)$ converge to $N_a(E)$ in the sense of Kuratowski.

Thanks to the geometric characterisation of minimisers using normal cones, we obtain Theorem 4 concerning the convergence of minimisers.

*Theorem 4*. Assume that the vectors $c_\nu \in \mathbb{R}^n$ converge to $c$ and let $M_\nu$ denote the set of minimisers of $f_\nu(x) = c_\nu^T x$ in $E = E_{A,b}$. The sequence $(M_\nu)$ converges then in the sense of Kuratowski to the set $M \subset E$ being the set of minimisers for the limiting functional $f(x) = c^T x$.

## 7. Conclusions

This paper presented the theorems which complete the formal description of Schumpeterian evolution of the Debreu model. We showed that if only the price system evolves to some final state and the innovations (determining the production sets $Y_j$) are such that the sequence $(Y_j)$ is convergent in the Kuratowski sense to some set $\tilde{Y}$, then the producers in the final production system are not only better off, but they achieve the maximal possible profit. It is also possible to approach the plan providing this maximal profit by a sequence of optimal production plans in the process of evolution. The character of Kuratowski convergence has additional practical implications: a numerical scheme (simplex method) applied to this problem will provide the result as optimum or close to optimum (due to possible numerical errors).

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**Optima producentów w ewolucji Schumpetera**
(Streszczenie)

W artykule rozważany jest system produkcji w modelu równowagi ogólnej Debreu. Zgodnie z teorią J. Schumpetera w celu wybicia gospodarki ze stagnacji konieczne jest wprowadzenie innowacji. W ich wyniku otrzymuje się ciągi planów optymalnych, odpowiadających każdemu etapowi wprowadzania innowacji. W pracy scharakteryzowano proces zmiany optymalnych planów produkcji i opisano warunki, przy których zagwarantowana jest ich zbieżność w taki sposób, że stan graniczny realizuje optimum stanu finalnego.

**Słowa kluczowe:** model Debreu, system produkcji, ewolucja Schumpetera, innowacje, zbieżność Kuratowskiego.